

*Student-Teacher  
Friendly Guide:  
Keep this handy  
for test prep!*

# Mastering Exponents & Polynomials

## From Basic Building Blocks to ACT Mastery

*Includes  
specific tips  
for ACT Math  
sections.*

# The Foundation: Understanding Exponents

Exponents represent repeated multiplication.

Base  $\rightarrow$   $2^5 = 32$  Exponent

$$2 \times 2 \times 2 \times 2 \times 2 = 32$$

**WARNING:**  
 $2^5$  is NOT  $2 \times 5$ .  
It is 2 multiplied  
by itself 5 times!

The little number  
(exponent) tells the  
big number (base)  
how many times to  
multiply itself.

# The Big Three Rules

## 1. Product Rule

$$x^m \cdot x^n = x^{m+n}$$

$$x^3 \cdot x^2 = x^5$$

## 2. Quotient Rule

$$\frac{x^m}{x^n} = x^{m-n}$$

$$\frac{x^6}{x^2} = x^4$$

## 3. Power Rule

$$(x^m)^n = x^{mn}$$

$$(x^2)^4 = x^8$$

Shortcut:  
Multiplying bases?  
ADD exponents.  
Dividing bases?  
SUBTRACT exponents.

For Power  
of a Power,  
parentheses  
mean MULTIPLY.

# Edge Cases & Special Rules

## The Zero Exponent

$$x^0 = 1$$

$$5^0 = 1$$

**CRITICAL:**  
Anything to the  
power of 0 is 1,  
NOT 0!

## The Negative Exponent

$$x^{-n} = \frac{1}{x^n}$$

$$x^{-3} = \frac{1}{x^3}$$

Negative exponents  
are just fractions  
in disguise. They  
flip the term to  
the denominator.

# Anatomy of a Polynomial

Etymology:  
Poly = "Many",  
Nomial =  
"Terms/Names"

Degree (Highest Exponent)

Constant Term

$$3x^2 + 5x - 7$$

Coefficient

Variable

Standard Form = Arranged from highest degree to lowest.

ACT TIP: The ACT often asks for the "Degree"—just look for the biggest exponent number.

# Valid vs. Invalid: Spot the Impostors

✓ Valid Polynomials

✗ NOT Polynomials

$$2x + 3 \text{ (Linear)}$$

$$x^2 + 4x + 4 \text{ (Quadratic)}$$

$$x^3 - 2x^2 + x \text{ (Cubic)}$$

*Rule of Thumb:  
No variables  
downstairs  
(denominator)  
and no negative  
exponents allowed!*

$$\frac{3}{x} + 2 \text{ (Variable in denominator)}$$

$$x^{-2} + 5 \text{ (Negative exponent)}$$

$$x^{\frac{1}{2}} + 3 \text{ (Fractional exponent)}$$

# Adding & Subtracting: Combining Like Terms

Think "Apples  
to Apples."  
You can't add  
an  $x^2$  to an  $x$ .

## Section 1: Addition

$$(3x^2 + 2x + 5) + (x^2 - 4x + 3)$$



$$4x^2 - 2x + 8$$

## Section 2: Subtraction

$$(5x^2 + 3x - 2) - (2x^2 + x + 4)$$

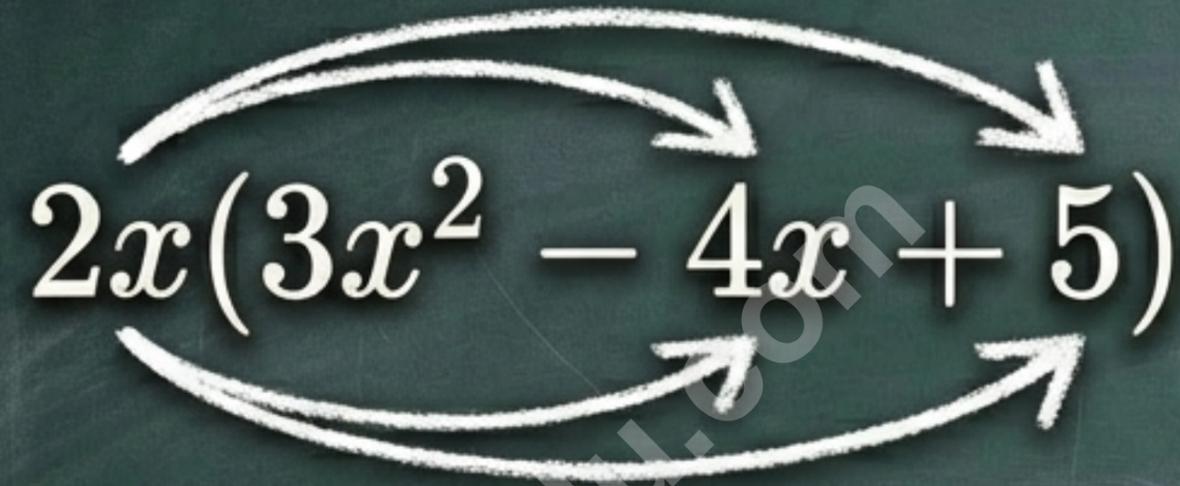
$$5x^2 + 3x - 2 - 2x^2 - x - 4$$



$$3x^2 + 2x - 6$$

**Subtraction  
Danger Zone:**  
You **MUST** distribute  
the negative sign to  
**EVERY** term in the  
second group.

# Multiplying: The Distributive Property

$$2x(3x^2 - 4x + 5)$$


- $2x \cdot 3x^2 = 6x^3$
- $2x \cdot -4x = -8x^2$
- $2x \cdot 5 = 10x$

$$6x^3 - 8x^2 + 10x$$

Don't forget the Product Rule:  
When multiplying terms, add their exponents  
( $x^1 \cdot x^2 = x^3$ ).

# Multiplying Binomials: The FOIL Method

$$(x + 3)(x + 5)$$

**First:**  $x \cdot x = x^2$

**Outer:**  $x \cdot 5 = 5x$

**Inner:**  $3 \cdot x = 3x$

**Last:**  $3 \cdot 5 = 15$

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$$x^2 + 8x + 15$$

FOIL is just a fancy way to say "Double Distribution"—everything in the first group multiplies everything in the second.

# Special Products (Shortcuts)

## Difference of Squares

$$(a + b)(a - b) = a^2 - b^2$$

$$(4x + 7)(4x - 7) = 16x^2 - 49$$

ACT TIP:  
Memorize the  
"Difference of  
Squares" pattern!  
It appears on  
almost every  
standardized test.

## Perfect Square Trinomials

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(x + 3)^2 = x^2 + 6x + 9$$

# Dividing Polynomials

## Monomial Division

$$(6x^3 + 9x^2) \div 3x$$

$$\frac{6x^3}{3x} + \frac{9x^2}{3x}$$

$$2x^2 + 3x$$

## Long Division Concept

$$(x^2 + 5x + 6) \div (x + 2)$$

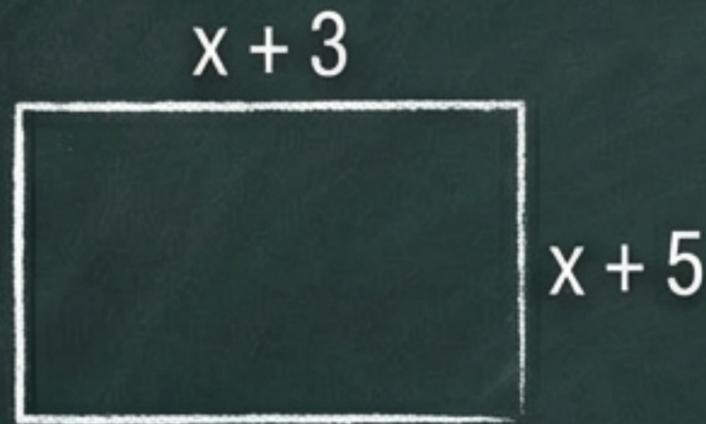
1. Divide leading terms
2. Multiply & Subtract
3. Repeat

$$x + 3$$

Long Division Mnemonic:  
Dad, Mom, Sister,  
Brother  
→ Divide, Multiply,  
Subtract,  
Bring down.

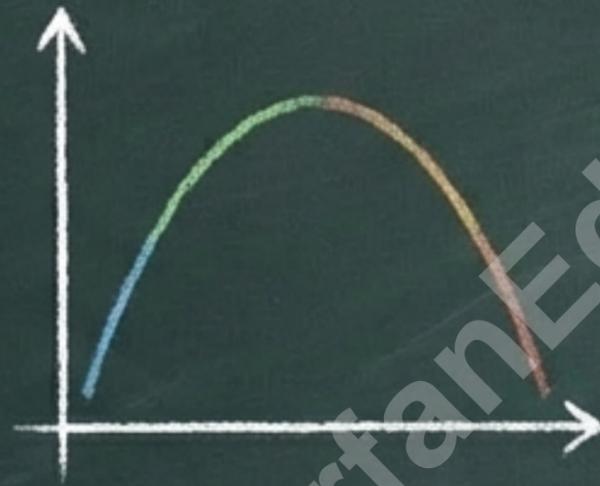
# Polynomials in the Real World

## Geometry



$$\text{Area} = x^2 + 8x + 15$$

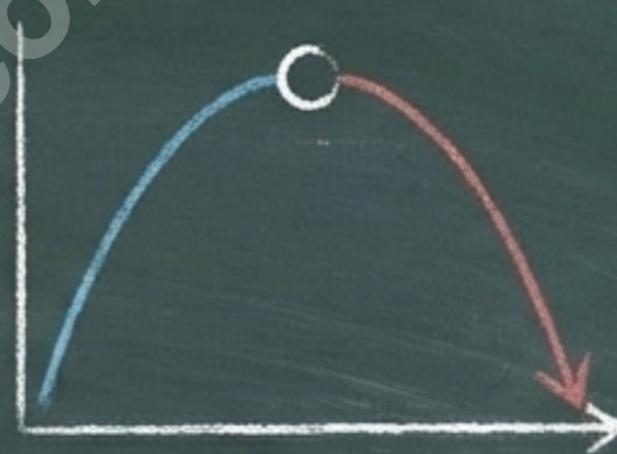
## Business



Profit Function:

$$P(x) = -2x^2 + 50x - 100$$

## Physics



Projectile Motion:

$$h(t) = -16t^2 + 64t + 80$$

Polynomials are excellent at modeling curves, from roller coasters to profit margins.

# The Exam Room Challenge (ACT Style)

If  $x^2 - y^2 = 48$  and  $x - y = 6$ , what is the value of  $x + y$ ?

1. Recognize Pattern:  $x^2 - y^2$  is Difference of Squares



2. Expand:  $(x + y)(x - y) = 48$



3. Substitute:  $(x + y)(6) = 48$



4. Solve: Divide by 6  $x + y = 8$

Don't just start calculating! Pause and look for the pattern ( $a^2 - b^2$ ) to solve this in 10 seconds.

# Quick Reference Guide

Product Rule	Add Exponents ( $x^a \cdot x^b = x^{a+b}$ )
Quotient Rule	Subtract Exponents ( $\frac{x^a}{x^b} = x^{a-b}$ )
Power Rule	Multiply Exponents ( $(x^a)^b = x^{ab}$ )
Zero Exponent	Always equals 1 ( $x^0 = 1$ )
FOIL	First, Outer, Inner, Last (Binomials)

Pro Tip:  
Screenshot this  
slide and use it  
while doing your  
homework  
practice problems.

# Class Dismissed!

You are now equipped to handle algebraic operations with confidence.

Math is a language.  
You just learned  
the grammar.  
Now go write  
some stories!

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