

Mastering Exponents & Roots

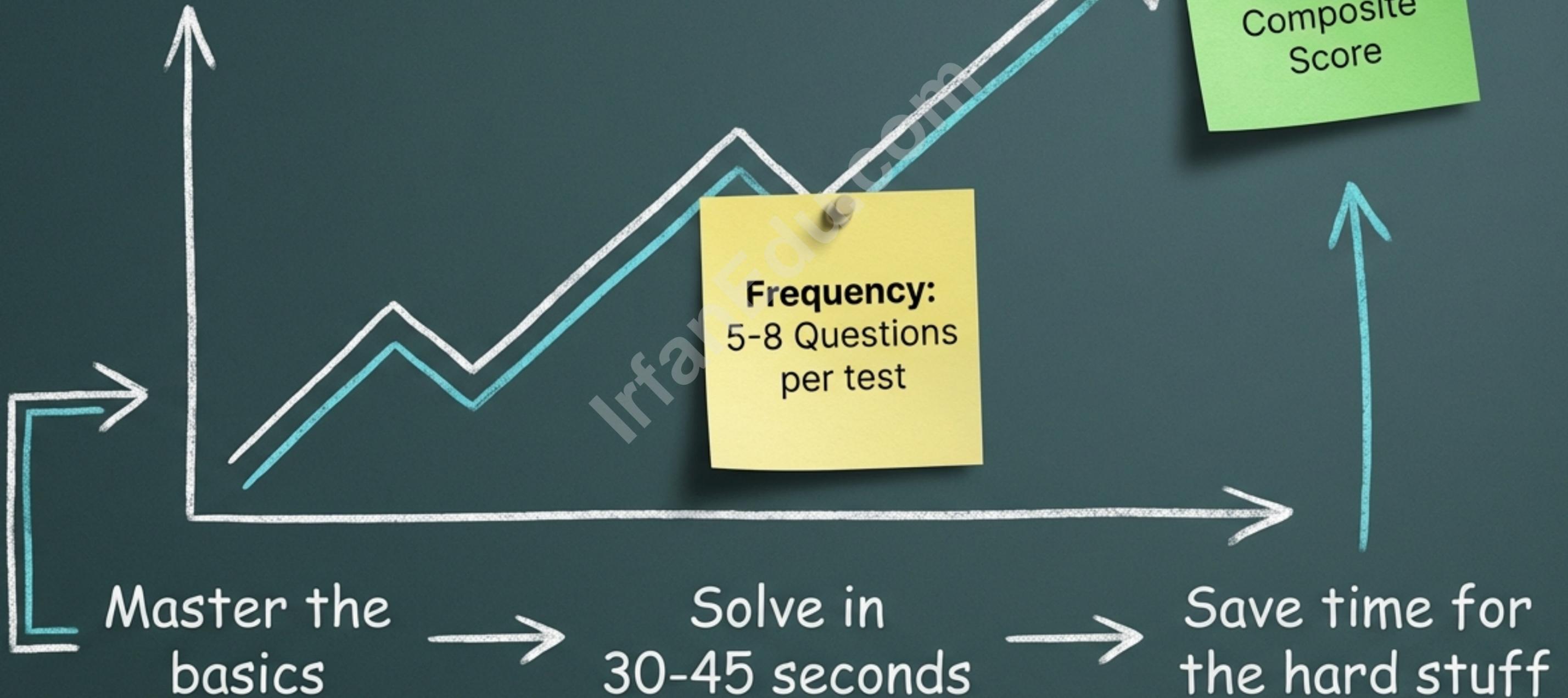
The ACT Math Score Booster

IrfanEdu.com

For Grades
9-12

Goal:
Simplify fast,
solve correctly

Why This Matters



Exponents:
Repeated Multiplication

Roots:
The Inverse Operation

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Inverse
Operations

$$\sqrt[3]{8} = ?$$

What number
x itself 3
times = 8?

The Core Laws: Multiplying & Dividing

Product Rule

$$a^m \cdot a^n = a^{m+n}$$

ADD the exponents.

$$x^3 \cdot x^5 = x^8$$

Quotient Rule

$$\frac{a^m}{a^n} = a^{m-n}$$

SUBTRACT the exponents.

$$\frac{y^7}{y^3} = y^4$$

The Core Laws: Power Plays

Power Rule

$$(a^m)^n = a^{m \cdot n}$$

MULTIPLY the exponents.

Pro Tip: Don't confuse this with the Product Rule!

Power to a power means multiply.

$$(z^2)^4 = z^8$$

The Weird Ones

Zero Rule:

$$a^0 = 1$$

NOT 0! Any non-zero number to the zero power is 1.

$$5^0 = 1$$
$$(xyz)^0 = 1$$

Negative Rule:

$$x^{-n} = \frac{1}{x^n}$$

Negative exponent?
FLIP IT.

$$x^{-3} = \frac{1}{x^3}$$

Root Operations

Square Root

$$\sqrt{16} = 4 \text{ (since } 4^2 = 16\text{)}$$

Cube Root

$$\sqrt[3]{27} = 3 \text{ (since } 3^3 = 27\text{)}$$

Root Product Rule

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

Multiply under the same radical.

$$\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$$

Anatomy of a Complex Problem

Step 1: Distribute
Power $\rightarrow 8x^9y^6$

$$((2x^3y^2)^3) \cdot x^4$$

Step 2: Combine Top
Terms $\rightarrow 8x^{13}y^6$

$$4x^5y^2$$

Step 3: Quotient Rule
(Subtract)

$$2x^8y^4$$

Taming the Radical

Simplify $\sqrt{72}$

The Pro Way

Recognize Perfect Square: $36 \cdot 2$

$$\sqrt{36} \cdot \sqrt{2}$$

$$\boxed{6\sqrt{2}}$$

The Safe Way

Factor Tree: $4 \cdot 18 \rightarrow 4 \cdot 9 \cdot 2$

$$2 \cdot 3 \cdot \sqrt{2}$$

$$\boxed{6\sqrt{2}}$$

Perfect Squares to Memorize:

4, 9, 16, 25, 36, 49, 64, 81, 100, 144

Don't Do This! (Common Mistakes)

Adding when you should multiply.

$$\begin{aligned} \cancel{(x^2)^3} &\neq x^5 \\ &= x^6 \end{aligned}$$

Distributing over addition.

$$\begin{aligned} \cancel{(x+y)^2} &\neq x^2 + y^2 \\ &= x^2 + 2xy + y^2 \end{aligned}$$

The Zero Trap.

$$\begin{aligned} \cancel{5^0} &\neq 0 \\ &= 1 \end{aligned}$$

Quick Drill

Q: Equivalent
to $\frac{x^8}{x^3}$?

Ans: x^5 (Subtract!)

Q: Value of
 $(3^2)^3$?

Ans: 729 (Multiply
exponents: 3^6)

Q: Simplify
 $\sqrt{50}$.

Ans: $5\sqrt{2}$ (Factor $25 \cdot 2$)

ACT Pro-Tips (Speed Strategy)

Memorize **Squares**
up to 12^2 & Cubes
up to 5^3 .

$$1^2=1, 2^2=4, \dots, 12^2=144;$$
$$1^3=1, 2^3=8, \dots, 5^3=125$$

Calculator Strategy:
Use for numbers (7^4),
NOT algebra (x^7).
It cannot simplify
variables for you.



→ 2401

→ ~~(x^7)~~

Test Small Numbers:
If stuck on a rule,
plug in $x=2$ to check
your math. Does the
result match?

Is $(x^2)^3 = x^5$?

Check with $x=2$:

$$(2^2)^3 = 4^3 = 64$$

$$2^5 = 32.$$

64 ≠ 32. No!

Time Management Strategy



- Easy Qs: 30-45 seconds
- Hard Qs: 90 seconds max

The 30-Second Rule: If you're stuck on a simplification for $>30s$, mark it and move on. Return with fresh eyes.

Strategic Guessing: Eliminate answers with obvious errors.

The Master Cheat Sheet

Product Rule:

$$x^a \cdot x^b = x^{a+b}$$

Quotient Rule:

$$\frac{x^a}{x^b} = x^{a-b}$$

Power Rule:

$$(x^a)^b = x^{ab}$$

Zero Rule:

$$x^0 = 1$$

Negative Rule:

$$x^{-n} = \frac{1}{x^n}$$

Roots:

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Frac Exp:

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

You've Got This!

Consistent practice with focused strategy beats cramming every time.

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For more topics (Polynomials, Scientific Notation), visit IrfanEdu.com.

