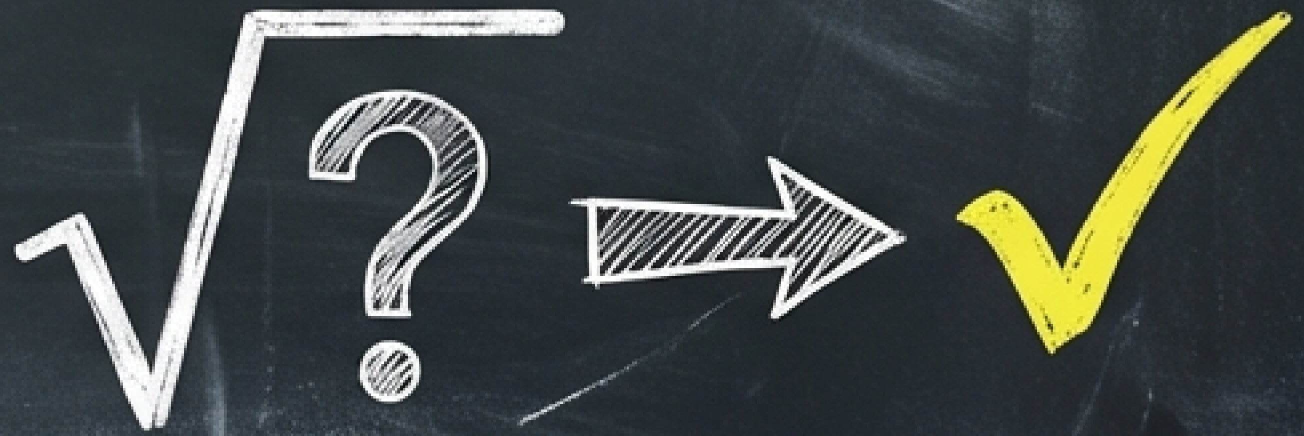
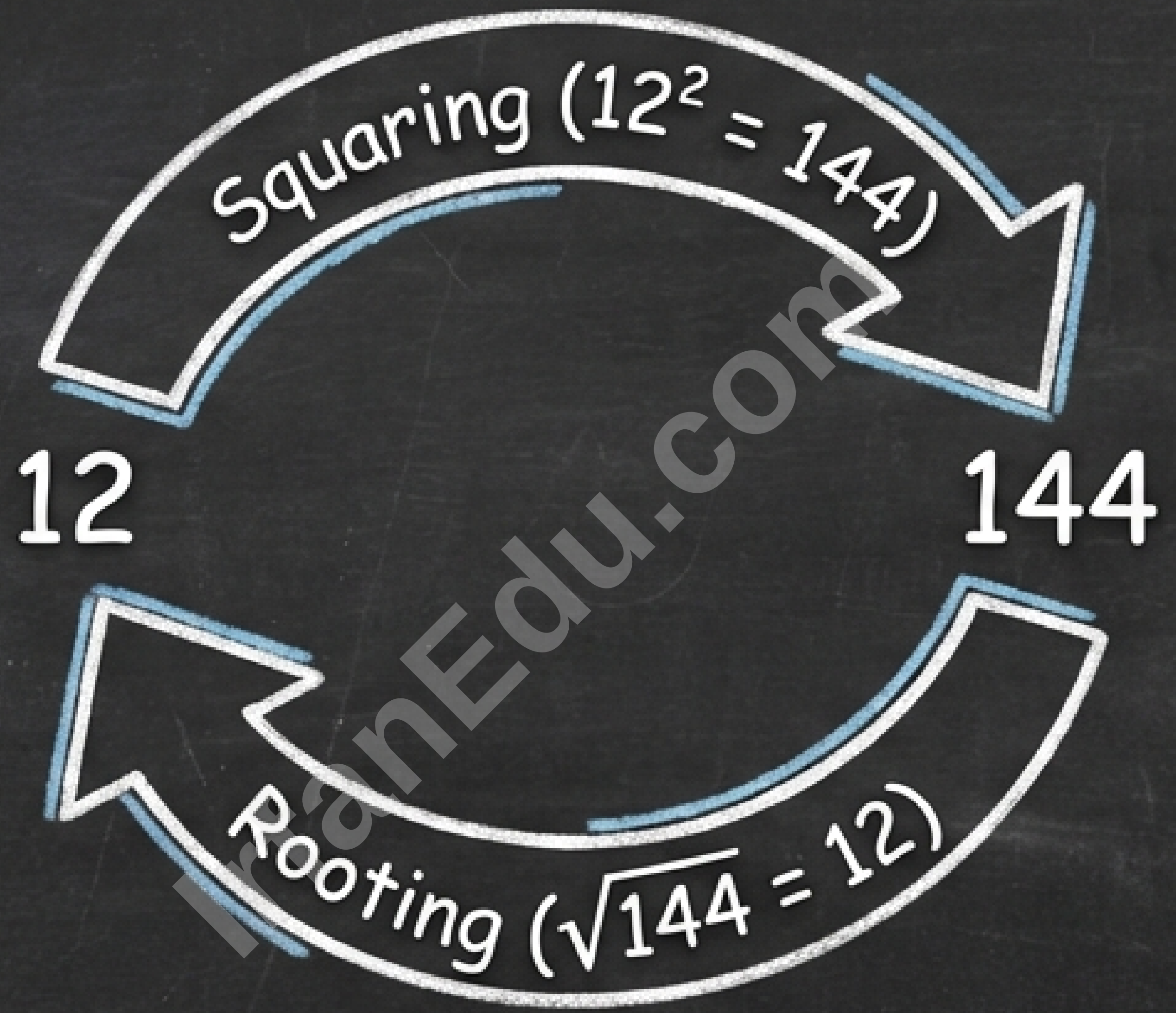
Mastering Radical Simplification

A Complete Guide to Square Roots

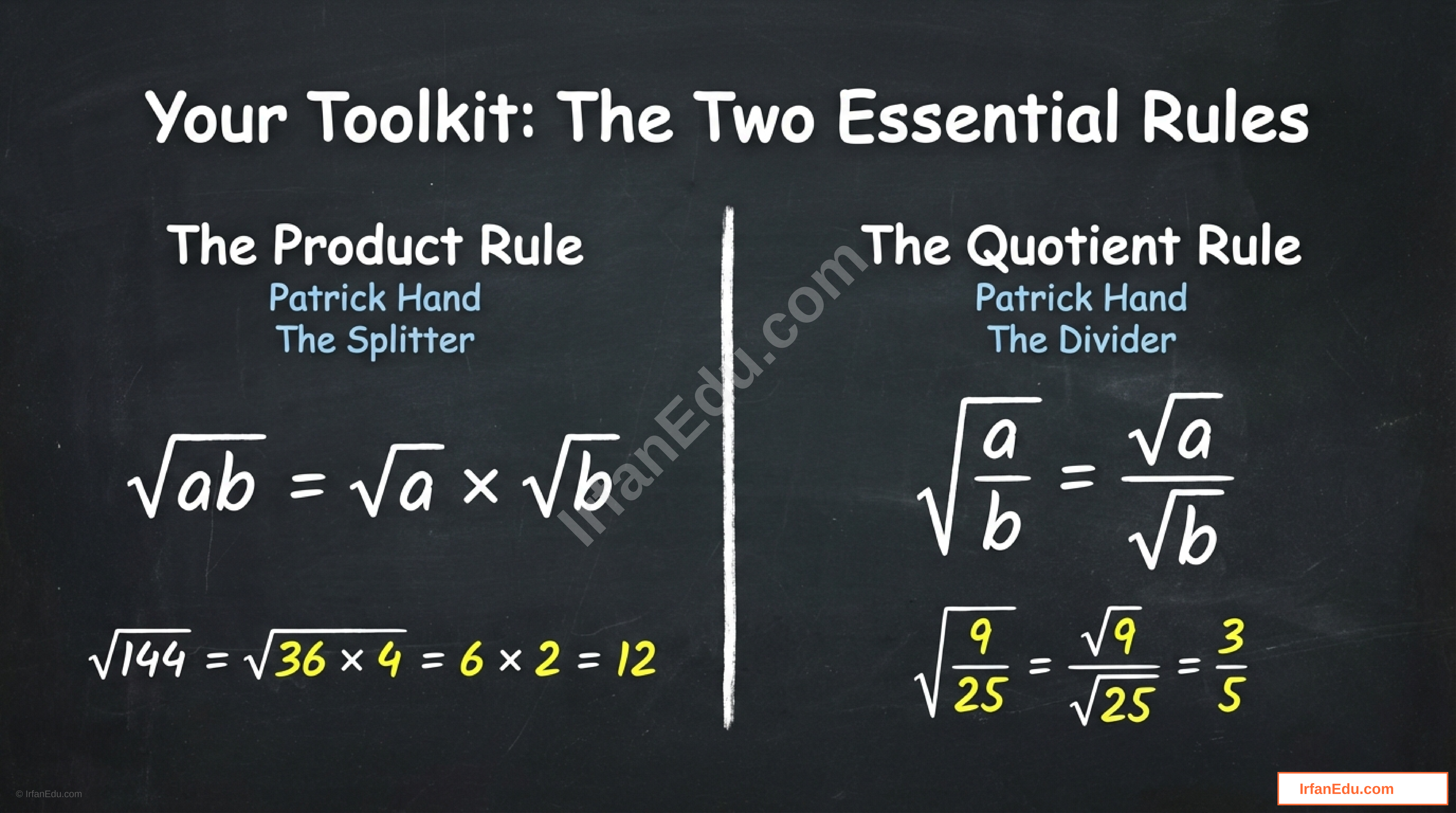


Roots and Powers are Inverse Operations



Radicals and exponents undo each other.
To simplify a root, find the number that was squared to create it.

$2^2 = \boxed{4} \Rightarrow \sqrt{4} = 2$ $\qquad$ $3^2 = \boxed{9} \Rightarrow \sqrt{9} = 3$


Your Toolkit: The Two Essential Rules
The Product Rule
Patrick Hand
The Splitter
$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$\sqrt{144} = \sqrt{36 \times 4} = 6 \times 2 = 12$
The Quotient Rule
Patrick Hand
The Divider
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$
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The Secret Weapon: Perfect Squares

Simplification is a hunt for these numbers. Memorize them up to 15.

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$

The Strategy: Find the Largest Perfect Square

$$\sqrt{450}$$

The Long Way

$$\sqrt{25 \times 18}$$

$$5\sqrt{18}$$

Not finished!
18 still has a factor of 9.

The Pro Way

$$\sqrt{225 \times 2}$$

$$15\sqrt{2}$$

Done in one step.

> **Pro Tip:** Identify the largest perfect square factor from the start to save time.

When is it Simplified?

☑ A radical is in simplest form ONLY when the radicand (number inside) contains no perfect square factors.

$$\sqrt{18}$$

✗

Contains 9 (a perfect square).

$$3\sqrt{2}$$

✓

2 has no square factors.

It's not just about making the number smaller; it's about extracting all "square-able" value.

Level 1: Basic Decomposition

Patrick Hand

Simplify $\sqrt{24}$

$$\sqrt{24}$$

$$\sqrt{4 \times 6}$$

We found the perfect square!

$$\sqrt{4} \times \sqrt{6}$$

$$2\sqrt{6}$$

Level 2: The Iterative Process

Simplify $\sqrt{72}$

$\sqrt{72}$

Direct Hit

$\sqrt{36 \times 2}$

$\downarrow$

$6\sqrt{2}$

scenic Route

$\sqrt{9 \times 8}$

$\downarrow$

$3\sqrt{8}$

Wait! 8 has a factor of 4

$\downarrow$

$3 \times \sqrt{4 \times 2}$

$\downarrow$

$3 \times 2\sqrt{2}$

$6\sqrt{2}$

Patrick Hand

Didn't spot 36? No problem. Just keep factoring until no squares remain.

Level 3: Handling Negatives

Patrick Hand

Simplify $-\sqrt{288}$

$$-\sqrt{288}$$

$$-(\sqrt{144} \times 2)$$

$$-(12\sqrt{2})$$

$$-12\sqrt{2}$$

The negative sign is a passenger—it stays outside the car while you drive.

Level 4: Radical Fractions

Simplify $\sqrt{75/4}$

$$\sqrt{\frac{75}{4}}$$

$$= \frac{\sqrt{75}}{\sqrt{4}}$$

$$= \frac{\sqrt{25 \times 3}}{2}$$

$$= \frac{5\sqrt{3}}{2}$$

Apply the Quotient Rule first. Divide and conquer.

Level 5: Binomials & Division

$$\frac{(3+\sqrt{18})}{3}$$

Simplify $\sqrt{18} \rightarrow 3\sqrt{2}$

Rewrite equation as $\dfrac{3+3\sqrt{2}}{3}$

Factor out the 3 in the top $\dfrac{3(1+\sqrt{2})}{3}$

$$= \frac{\cancel{3}(1+\sqrt{2})}{\cancel{3}}$$

Final Result: $1+\sqrt{2}$

Warning: Do NOT cancel the 3s yet! You must simplify the radical first.

The Principal Root Rule

Expression

$\sqrt{4}$

$= 2$

Refers to the positive (principal) root only.

Equation

$x^2 = 4$

$x = 2$ or $x = -2$

Two possible answers.

Unless there is a $\pm$ sign in front of the root, assume positive.

Non-Perfect Squares: Exact vs. Approximate

Not every number escapes the radical.
$\sqrt{3}$ has no perfect square factors.

$$\boxed{\sqrt{3}} \rightarrow \neq \leftarrow \boxed{\approx 1.732}$$

Exact Answer.
(Use for Math Class).

Approximation.
(Use for Building a Bridge).

Summary: Your Rules of Engagement

1. **Memorize the Grid:** Know your perfect squares up to 15^2.
2. **Hunt Big:** Look for the largest factor first to save steps.
3. **Check Your Work:** Ensure the remainder has no square factors left.
4. **Respect the Sign:** Keep negatives outside; recognize √ means positive.
5. **Use the Tools:** Split products and quotients when stuck.

Radicals aren't random. They are puzzles waiting to be solved.

You now have the tools to break them down.
Go find the perfect squares.

Class Dismissed √☺)