

Mastering Direct & Inverse Proportions

ACT Math Guide for Grades 9–10

The topic that adds 3–4 points to your composite score.

Based on the guide by Irfan Mansuri, ACT Test Prep Specialist.

Why This Matters for the ACT

- ★ **Frequency:** Appears in 5–8 questions per test section.
- ★ **Impact:** Mastery can add 3–4 points to your final composite score.
- ★ **Versatility:** Used in word problems, geometry (similar triangles), and rate conversions.

Mastering this gives you a versatile tool for tackling multiple question types quickly.

34 A+

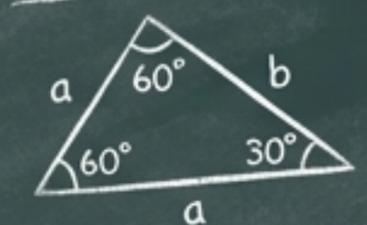
1) $\frac{3}{12} + \frac{3}{24} = \frac{5}{20}$

2) $\frac{5x^3}{x} - \left(\frac{1}{2}\right)^2 = \frac{1}{3}$

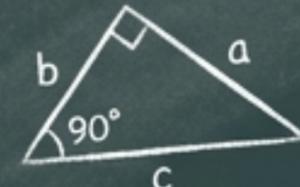
3) $\frac{x}{30} - \frac{6}{20} = \frac{30}{9}$

4) If a train travels at a $\frac{1}{h}$ m per hr 800
lsing/h? $y = \frac{3x - 4}{3 - 1}$
 $\frac{1x - 3x}{6x} = \frac{25 \text{ min}}{100}$

5) Find the value of x,
are caled.
 $\frac{x - 5}{x} = \frac{3 + 3}{10}$

5) 

6) Find the value of x



7) Rate rate and rate conversions.

$$\frac{dx}{dx} = \frac{\ln(x - 210)}{(n - 1)}$$

The Foundation: Ratio vs. Proportion

Ratio

A comparison between two quantities.



2 cups flour : 1 cup sugar (2:1)

Proportion

An equation stating two ratios are equal.

$$\frac{a}{b} = \frac{c}{d}$$

This equation is the engine for solving real-world ACT problems.

Concept A: Direct Proportion

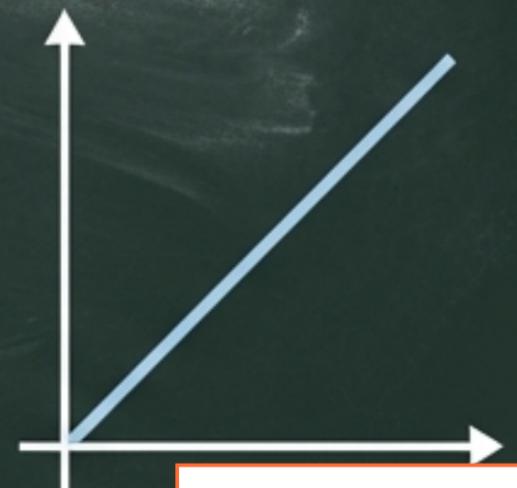


We move together.

When one quantity increases, the other increases proportionally.

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

Alternate: $x_1 \cdot y_2 = x_2 \cdot y_1$
(Cross-Multiplication)



Cost & Quantity • Recipe Scaling • Distance & Time (constant speed)

Direct Example: The Apple Problem

If 5 pounds of apples cost \$8.75, how much will 12 pounds cost?



Step 1. Logic: More pounds = More cost. (Direct)

Step 2.
$$\frac{5}{8.75} = \frac{12}{x}$$

Step 3:
$$\frac{5}{8.75} = \frac{12}{x}$$

Step 3. Cross-multiplication

$$\frac{5}{8.75} = \frac{12}{x}$$

$$5 \cdot x = 8.75 \cdot 12$$

$$5x = 105$$

$$x = 21 \text{ (Answer: \$21.00)}$$

Concept B: Inverse Proportion (Caveat Brush)

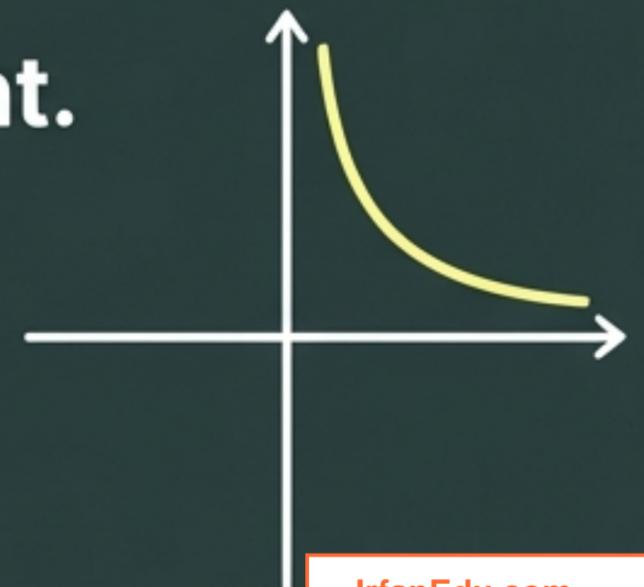


We move opposite.

When one quantity increases, the other decreases proportionally.

$$x1 \cdot y1 = x2 \cdot y2$$

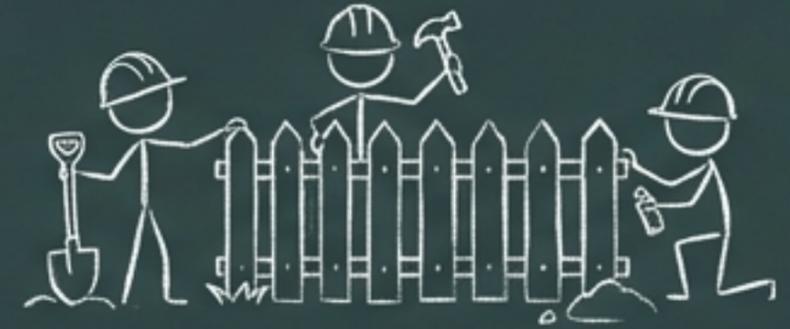
The product remains constant.
Do **NOT** cross-multiply here.



Workers & Time • Speed & Time (fixed distance)

Inverse Example: The Worker Problem

6 workers take 8 hours to build a fence.
How long for 4 workers?



Step 1: Logic: Fewer workers = MORE time. (Inverse)

$$6 \text{ workers} \times 8 \text{ hours} = 4 \text{ workers} \times t$$

Step 2: $48 = 4t$

Step 3: $t = 12 \text{ hours}$

Logic Check: Does it make sense? Yes. Fewer workers (6 \rightarrow 4) means it takes longer (8 \rightarrow 12).

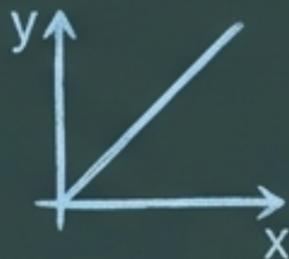
The Cheat Sheet: Direct vs. Inverse

DIRECT



$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

DIVIDE to find constant ratio.

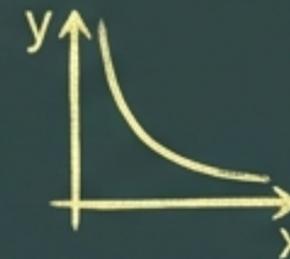


INVERSE



$$x_1 \cdot y_1 = x_2 \cdot y_2$$

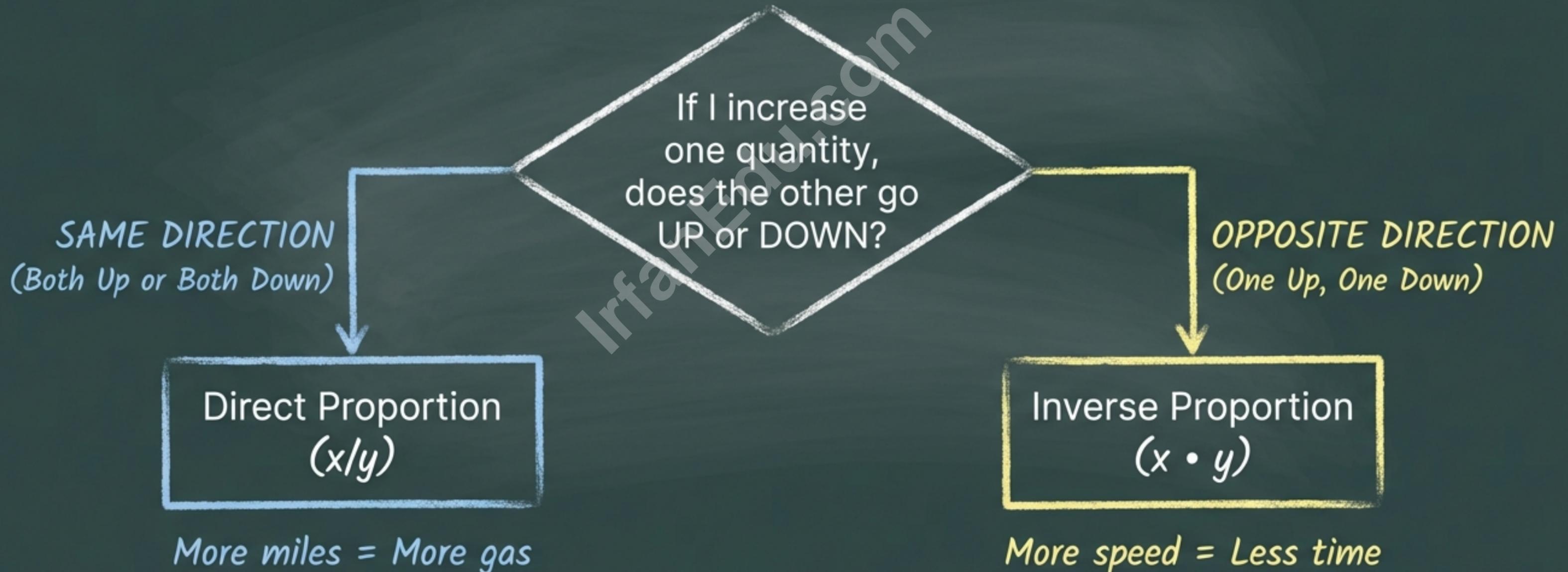
MULTIPLY to find constant product.



IrfanEdu.com

Strategy #1: The Direction Test

Before calculating, ask yourself...



Strategy #2: The Unit Rate Shortcut



Instead of setting up a full equation, find the value of ONE unit first.

The Apple Problem Revisited

$$5 \text{ lbs} = \$8.75$$



$$\frac{\$8.75}{5} = \$1.75 \text{ (Price per ONE pound)}$$

Multiply



$$\$1.75 \times 12 = \$21.00$$

This is often **faster and less error-prone** than cross-multiplying on the calculator.

Warning: Common Mistakes



The Wrong Formula

Using x/y for an inverse problem.

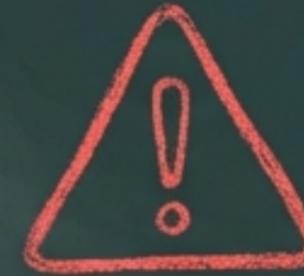
Fix: Always use the Direction Test first.



The Bad Setup

Mixing units (e.g. Dollars/Pounds = Pounds/Dollars).

Fix: Write the unit words out next to the numbers.



Arithmetic Errors

Rushing the cross-multiplication.

Fix: Use your calculator strategically.

The Reasonableness Check



Always verify the logic before moving to the next question.

Scenario: If you calculated that 4 workers would take 2 hours (instead of 12)...

STOP.

Fewer workers cannot build a fence faster.
If inputs go down in an inverse problem,
the output **MUST** go up.

Takes 5 seconds. Saves the point.

Practice: The Factory Problem

Difficulty: Intermediate

8 machines produce 240 units in 6 hours. If 2 machines break down, how long for the remaining machines to produce the same units?

- → **Identify:** Fewer machines = More time. (Inverse Proportion)
- → **Calculate Remaining:** $8 - 2 = 6$ machines
- → **Setup:** $8 \text{ machines} \times 6 \text{ hours} = 6 \text{ machines} \times t$
- → **Solve:** $48 = 6t$
- → **$t = 8$ hours**

ACT Pro-Tips Summary

- ★ Direction Test: Up/Up = Direct. Up/Down = Inverse.
- ★ Visual Cue: Direct looks like a fraction (x/y). Inverse looks like a product (xy).
- ★ Scale Problems: Maps and Blueprints are always Direct Proportion.
- ★ Write it Out: “5 lbs is to \$8.75...” prevents setup errors.
- ★ Time Management: Target 45–60 seconds per question.

Ready to Master ACT Math?

Keep practicing these patterns. Understanding the “Why” behind the formula is the key to those extra 3–4 points.

Written by Irfan Mansuri, ACT Test Prep Specialist.
Source: Mastering Direct and Inverse Proportions: ACT Math Guide.

