

ALGEBRAIC SIMPLIFICATION: A Blackboard Guide

Mastering the Distributive Property & Like Terms

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OBJECTIVE:

Transform chaos into order by applying two fundamental tools: Distribution and Combination.

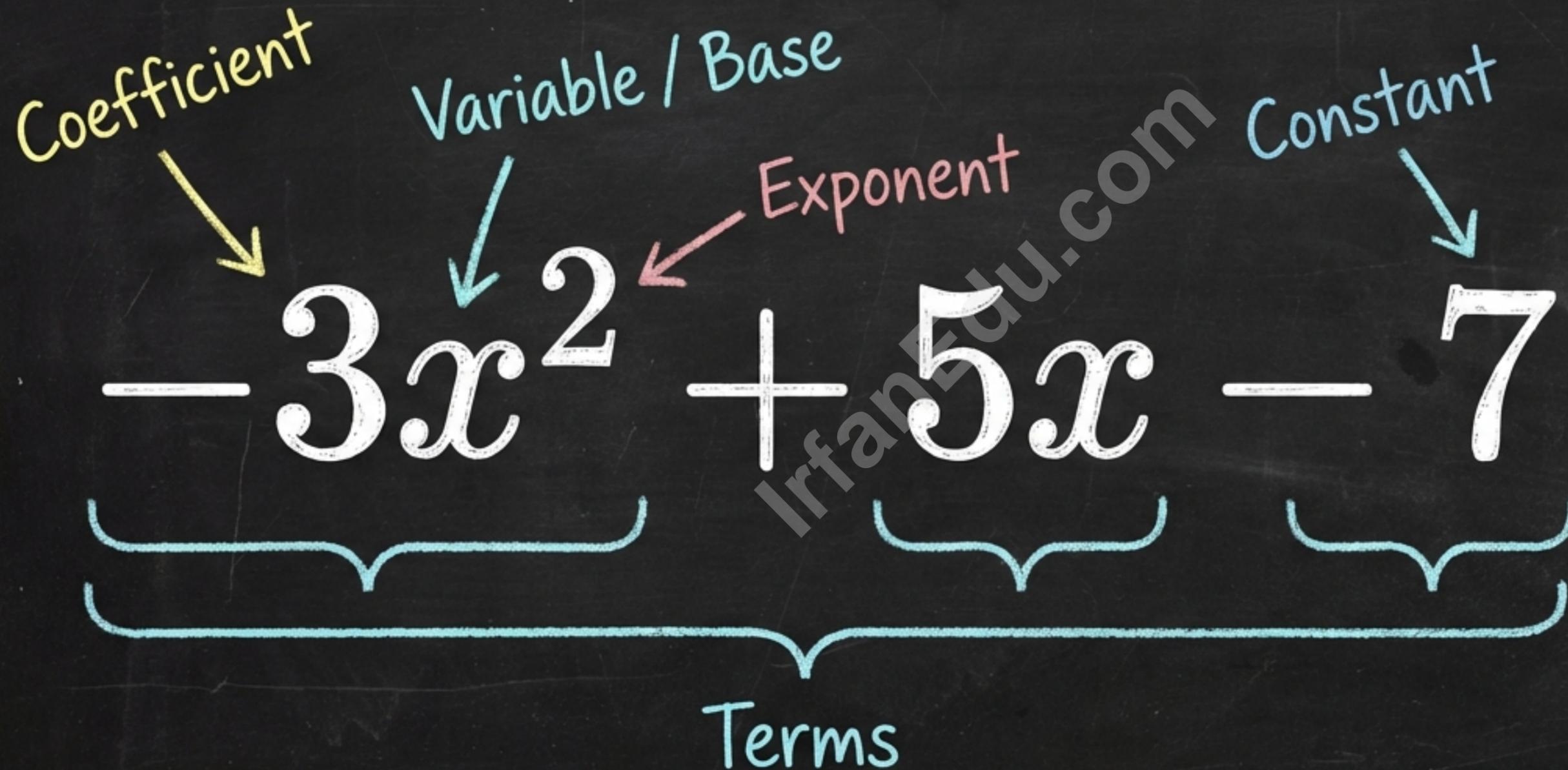
THE GOAL:

Reduce expressions to their simplest form $(ax + b)$ where no further operations are possible.

margin note

Vocabulary → Rules → Application → Verification

The Anatomy of an Expression



DEFINITIONS

Monomial:
One term ($5x$)

Binomial:
Two terms
($x + 5$)

Trinomial:
Three terms
($x^2 + 5x + 6$)

Polynomial:
One or more
terms

The Laws of the Board

Rule Name	Algebraic Definition	Concrete Example
Identity Property	$x + 0 = x$	$1 \cdot x = x$
Commutative Property	$a + b = b + a$	$5 \cdot x = x \cdot 5$
Associative Property	$(a + b) + c = a + (b + c)$	Grouping changes, sum stays same
Inverse Property	$5x + (-5x) = 0$	Opposites cancel out
Zero Property	$a \cdot 0 = 0$	Multiplication by zero destroys the term

Tool #1: The Distributive Property

$$a(b + c) = ab + ac$$

Mnemonic: Distribute the Gift
(Give the outside number to EVERYONE inside).

Worked Example: Problem: $5(7y + 2)$

Action: Multiply 5 by $7y$ AND 5 by 2.

Result: $35y + 10$

Reverse Process: Factoring Out
 $ab + ac = a(b + c)$

The Danger Zone: Distributing Negatives

Rule: Negative Changes Everything.

Safe

Danger

Positive Distribution

$$+(x + 3) \rightarrow x + 3$$

Simply remove parentheses.

Negative Distribution

$$-(x + 3) \rightarrow -x - 3$$

All signs flip.

Complex Example:

Problem: $-3(2x^2 + 5x + 1)$

$\rightarrow -6x^2 \rightarrow -15x \rightarrow -3$

Result: $-6x^2 - 15x - 3$

Tool #2: Identifying Like Terms

Like Terms: Terms with identical variable parts (same letters, same powers).

The Golden Rule: Like Attracts Like.

$$\textcircled{5x^2} + \boxed{3x} - \textcircled{4x^2} + \triangle 9y - \underline{\underline{7}}$$

Do Not Match

$$5x^2 \neq 3x$$

Different powers

$$7x \neq 7y$$

Different variables

The Art of Combining

$$ax + bx = (a + b)x$$

Add/Subtract coefficients.
Keep variables EXACTLY the same.

(Basic)

$$7x + 3x$$



$$(7 + 3)x$$



$$10x$$

(Subtle)

$$9y^2 - 4y^2$$



$$5y^2$$

(Multiple Variables)

$$5x^2y - 3xy^2 + 4x^2y - 2xy^2$$



$$(5 + 4)x^2y + (-3 - 2)xy^2$$



$$9x^2y - 5xy^2$$

The Simplification Checklist

Order of Operations (PEMDAS) applies strictly here.

Remove Parentheses
Use Distributive Property. Multiplication comes before Addition.

Identify Like Terms
Look for matching variables and exponents.

Combine
Add or subtract coefficients.

Standard Form
Arrange by highest power first (e.g., x^2 then x then constant).

*Pro-Tip: Multiplication before Addition applies to algebra too!
 $2 + 3(4)$ is 14, not 20.*

Worked Example: The Beast

$$2(3a - b) - 7(-2a + 3b)$$

Step 1: Distribute

Left: $2(3a) + 2(-b) \rightarrow 6a - 2b$

Watch Signs!

Right (Watch Signs!):

$$-7(-2a) - 7(3b) \rightarrow +14a - 21b$$

Step 2 & 3: Combine

Intermediate Line: $6a - 2b + 14a - 21b$

Group 'a' terms:

$$6a + 14a = 20a$$

Group 'b' terms:

$$-2b - 21b = -23b$$

Final Answer: $20a - 23b$

Critical Error: The '5 minus 2' Trap

$$5 - 2(x^2 - 4x - 3)$$

The Mistake

Incorrect Logic:
Subtracting $5 - 2$ first.

$$\cancel{3(x^2 - 4x - 3)}$$

The Solution

Correct Logic: Distribution
before Subtraction.

Keep the 5.
Distribute
the -2.

$$5 - 2x^2 + 8x + 6$$

$$\text{Final Result: } -2x^2 + 8x + 11$$

Advanced Distribution: Division & Fractions

Division is just multiplication by a reciprocal.

$$\text{Step 1: } (25x^2 - 5x + 10) \div 5$$

↓ Rewrite as:

$$\text{Step 2: } \left(\frac{1}{5}\right)(25x^2 - 5x + 10)$$

↓ Distribute $\left(\frac{1}{5}\right)$ to each term

$$\text{Step 3: } \frac{1}{5} \cdot 25x^2 \rightarrow 5x^2 \quad \left| \quad \frac{1}{5} \cdot -5x \rightarrow -x \quad \left| \quad \frac{1}{5} \cdot 10 \rightarrow 2$$

$$\boxed{5x^2 - x + 2} \text{ Final Result}$$

Fraction Coefficients

$$\frac{1}{2}a + \frac{3}{4}a = \frac{5}{4}a$$

← Combine numerators

Common Denominator (4)

Translation: Words to Algebra

Subtract $3x - 2$ from **twice the quantity** $(-4x^2 + 2x - 8)$

$$2(-4x^2 + 2x - 8)$$

$-(3x - 2)$ placed at the end

$$2(-4x^2 + 2x - 8) - (3x - 2)$$

1. Distribute 2: $-8x^2 + 4x - 16$

2. Distribute Negative: $-3x + 2$

3. Combine: $-8x^2 + x - 14$

Faculty Insights: Common Mistakes

Mistake vs. Correction Table

	Instinct	Reality
The Lazy Distribution	$3(x + 2) = 3x + 2$ ← Forgot second term	$3(x + 2) = 3x + 6$
Sign Blindness	$-2(x - 3) = -2x - 6$ ← Forgot to flip sign	$-2(x - 3) = -2x + 6$
Power Corruption	$3x^2 + 4x^2 = 7x^4$ ← Adding exponents incorrectly	$7x^2$ ← Variable part stays unchanged

Rapid Fire Practice

Level 1

Q: $-2(x + 1)$

A: $-2x - 2$

Level 2

Q: $6x^2y - 3xy^2 + 2x^2y - 5xy^2$

A: $8x^2y - 8xy^2$

Level 3

Q: $5x - 2(4x - 5)$

A: $-3x + 10$

Level 4

Q: $10 - 5(x^2 - 3x - 1)$

A: $-5x^2 + 15x + 15$

The Takeaway Cheatsheet

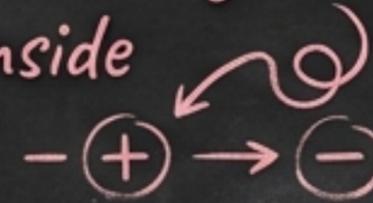
Distributive

Distribute the Gift.
Everyone inside gets 7 the outside number.



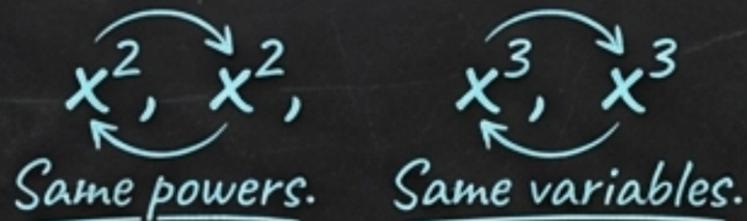
Negatives

Negative Changes Everything.
Flip ALL signs inside parentheses.



Like Terms

Like Attracts Like.
Same variables, same powers.



Combining

Variables Don't Change.
Only add/subtract coefficients.

$$\underline{2x} + \underline{3x} = \underline{5x}$$

Stays "x"

Practice makes perfect. Always check your work by substituting values.