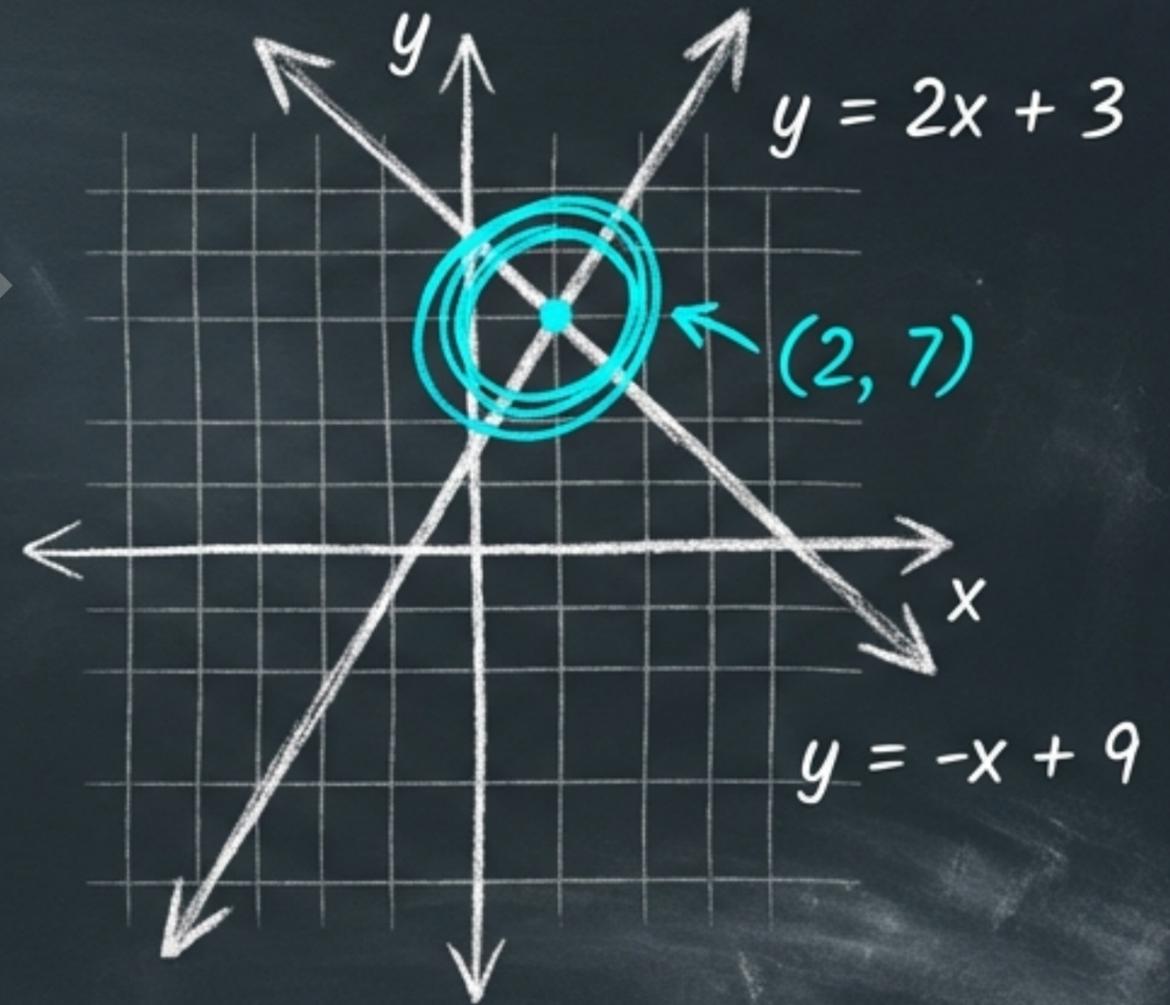


Mastering Systems of Equations

Finding the Balance Point



A Visual Guide to Solving Multiple Equations Simultaneously

What is a System?

A system of equations is a set of conditions that must all be true at the same time. It is not just about solving for x ; it is about finding the unique state where everything balances.

The Puzzle

$$A: x + y = 10$$

$$B: x - y = 4$$



The Solution:
 $x = 7, y = 3$

The Intersection Point

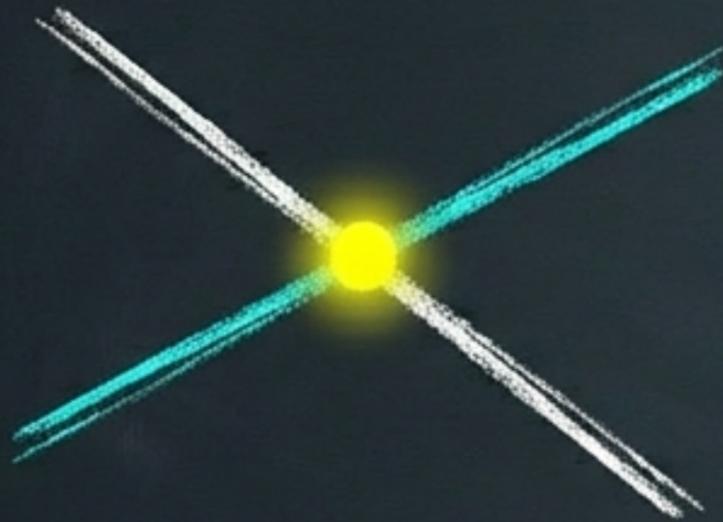
Check:

$$7 + 3 = 10 \text{ (True)}$$

$$7 - 3 = 4 \text{ (True)}$$

Visualizing the Outcome: Three Possibilities

One Solution



Lines intersect at exactly one point (x,y) . This is a consistent system. Look for different slopes.

No Solution



Lines are parallel and never meet. Inconsistent. Same slope, different y -intercept.

Infinite Solutions



Lines overlap completely. Identical equations. Same slope and same y -intercept.

Choose Your Method

Mastering both gives you strategic flexibility.

Method 1: Substitution

Best used when a variable is "lonely" or already isolated (e.g., $y = \dots$).

The "Plug It In" Strategy

Method 2: Elimination

Best used when variables are lined up vertically and coefficients match.

The "Stack and Cancel" Strategy

Deep Dive: The Substitution Method

Step-by-Step Breakdown

1. $y = 2x - 1$
2. $3x + y = 9$

$$y = 2x - 1$$

$$3x + y = 9$$

$$3x + (2x - 1) = 9$$

$$5x - 1 = 9$$

$$5x = 10$$

$$x = 2$$

Back-Substitute $x=2$ into top equation:

$$y = 2(2) - 1$$

$$y = 3$$

ACT Time Tip:
Use this when 'y'
or 'x' is already
by itself!
Target time: 60s.

Deep Dive: The Elimination Method

The Sandwich Technique

$$\begin{array}{r} 2x + 3y = 16 \\ + 5x - 3y = 5 \\ \hline \end{array}$$

Opposites Match!

$$7x + 0 = 21$$

$$x = 3$$

Plug $x = 3$ back in:

$$2(3) + 3y = 16$$

$$6 + 3y = 16$$

$$3y = 10$$

$$y = 10/3$$

ACT Time Tip:
Fastest method when coefficients are already opposites. Don't re-arrange if you don't have to!

Level up: Elimination with Multiplication

Force the Match

The Problem

$$3x + 2y = 12$$

$$4x - y = 5$$

Multiply bottom by 2

The Setup

$$3x + 2y = 12$$

$$8x - 2y = 10$$

The Solve

$$3x + 2y = 12$$

$$8x - 2y = 10$$

$$11x = 22$$

$$x = 2$$

$$\text{Solve for } y: 4(2) - y = 5 \rightarrow y = 3$$

$$(2, 3)$$

Real World Application: The Mixture Problem

The Reality (English)

A concert sold Adult tickets (\$25) and Student tickets (\$15).

Total tickets sold: 200.

Total revenue made: \$4,000.

Question: How many of each?

The Translation (Math)

Let a = Adult, s = Student

Equation 1 (Quantity): $a + s = 200$

Equation 2 (Value): $25a + 15s = 4000$

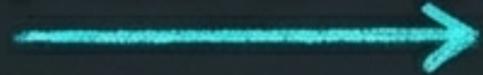
$$s = 200 - a \quad \curvearrowright \text{(Substitution)}$$

$$25a + 15(200 - a) = 4000$$

Result: 100 Adult, 100 Student.

Real World Application: Distance & Speed

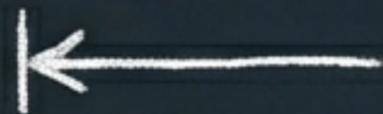
$$\text{Distance} = 60t$$



$$\text{Distance} = 45t$$



Car A
(60 mph)



Total Distance Needed = 75 miles



Car B
(45 mph)

Distance A + Distance B = Total

$$60t + 45t = 75$$

$$105t = 75$$

$$t = \frac{5}{7} \text{ hours (approx 43 mins)}$$

The ACT Strategy Guide

- Frequency: 3-5 questions per test

- Reward: +2 to 4 point score boost

Flexibility: Don't marry one method. If y is isolated, Substitute. If stacked, Eliminate.

The Back-Door: Stuck? Plug the multiple choice answers (x,y) back into the equations to see what works.

Check Work: Always ~~plug~~ tie the multiple choice answers (x,y) into the equations to see what works.

Check Work: Always back-substitute to verify. It takes 10 seconds and saves the point.

Pop Quiz: Test Your Skills

What is the solution to the following system?

$$x + y = 8$$

$$x - y = 2$$

A) (3, 5)

B) (5, 3)

C) (4, 4)

D) (6, 2)

E) (2, 6)



Pop Quiz: Solution

B) (5, 3)

The Math Way

$$\begin{array}{r} x + y = 8 \\ + (x - y = 2) \\ \hline \end{array}$$

$$2x = 10 \rightarrow x = 5$$

The Mental Check Way
Find two numbers that add to 8 and subtract to 2.

$$\text{Option B: } 5 + 3 = 8 \text{ (Yes)}$$

$$\text{Option B: } 5 - 3 = 2 \text{ (Yes)}$$

Done.

Summary Cheatsheet

- ☆ The Goal: Find the intersection point where ALL equations are true.
- ☆ Graphing: Visual check. Parallel = No Solution. Overlap = Infinite.
- ☆ Substitution: Use when a variable is 'lonely'. Plug it in.
- ☆ Elimination: Use when coefficients line up. Stack and cancel.
- ☆ Golden Rule: Plug your answer back in to check!



You've Mastered the System.

Whether graphing lines or solving word problems,
remember: you are just looking for the balance point.



Content adapted from IrfanEdu.com & ACT Math Guides.

