MASTERING FUNCTIONS & NOTATION

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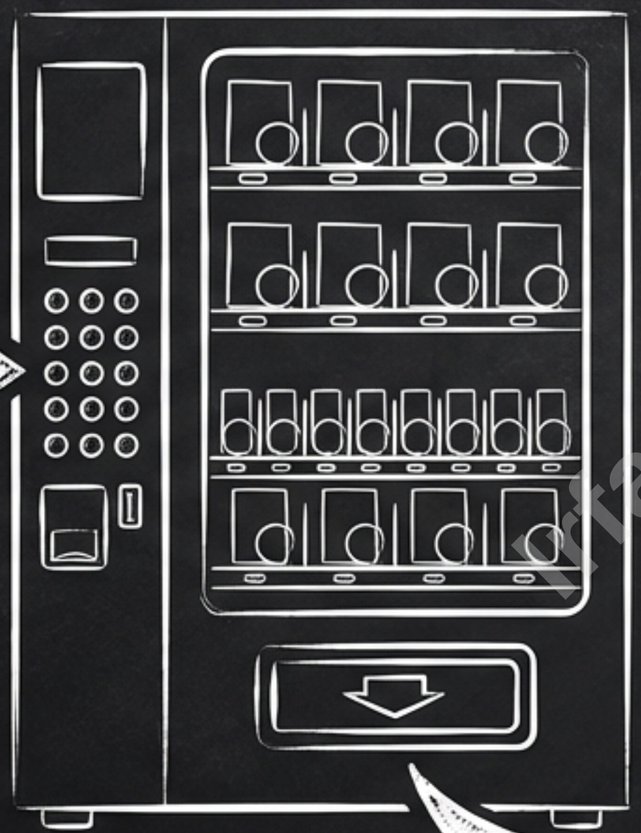
Basically: Math's way of predicting the future.

Courier Prime

A COMPREHENSIVE GUIDE TO MATHEMATICAL RELATIONSHIPS

Class: Mathematics 101

THE VENDING MACHINE PRINCIPLE



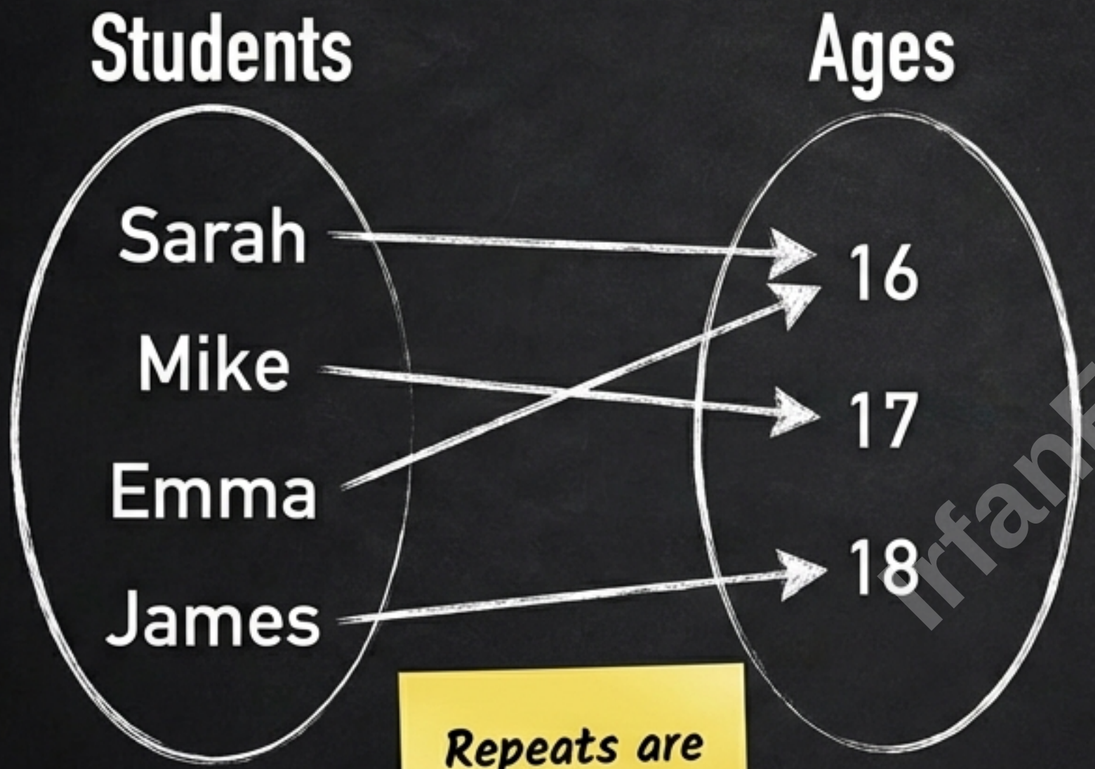
INPUT
(Button A3)

When you press A3,
you get Chips.
Every single time.

OUTPUT
(Chips)

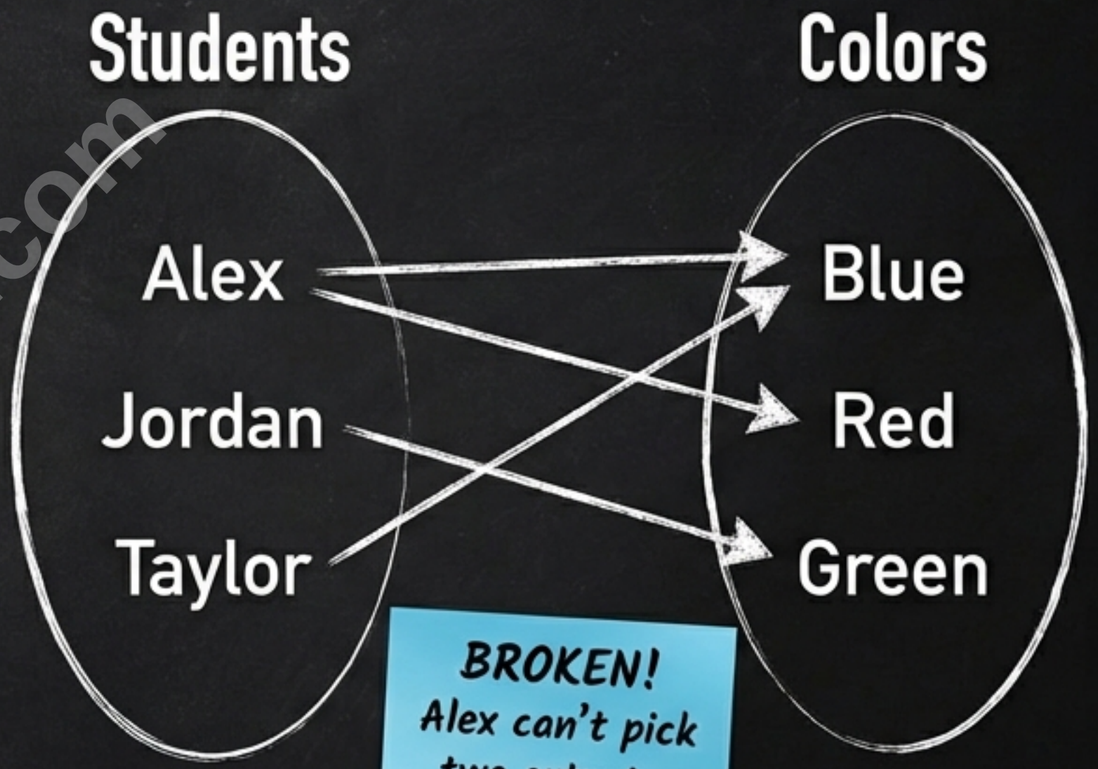
The Golden Rule:
You never press a button and get two different things.
No surprises!

FUNCTION (Students → Ages)



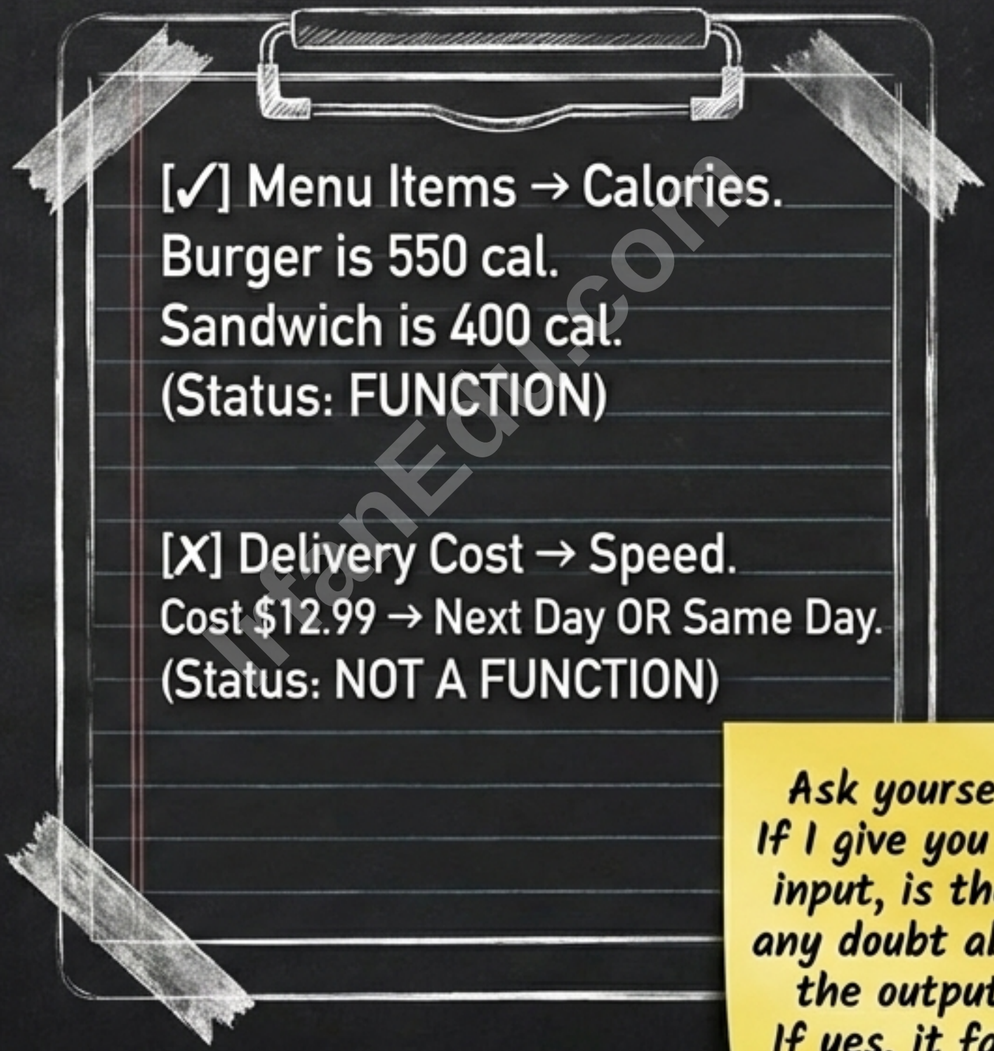
Repeats are okay here!
Sarah and Emma can both be 16.

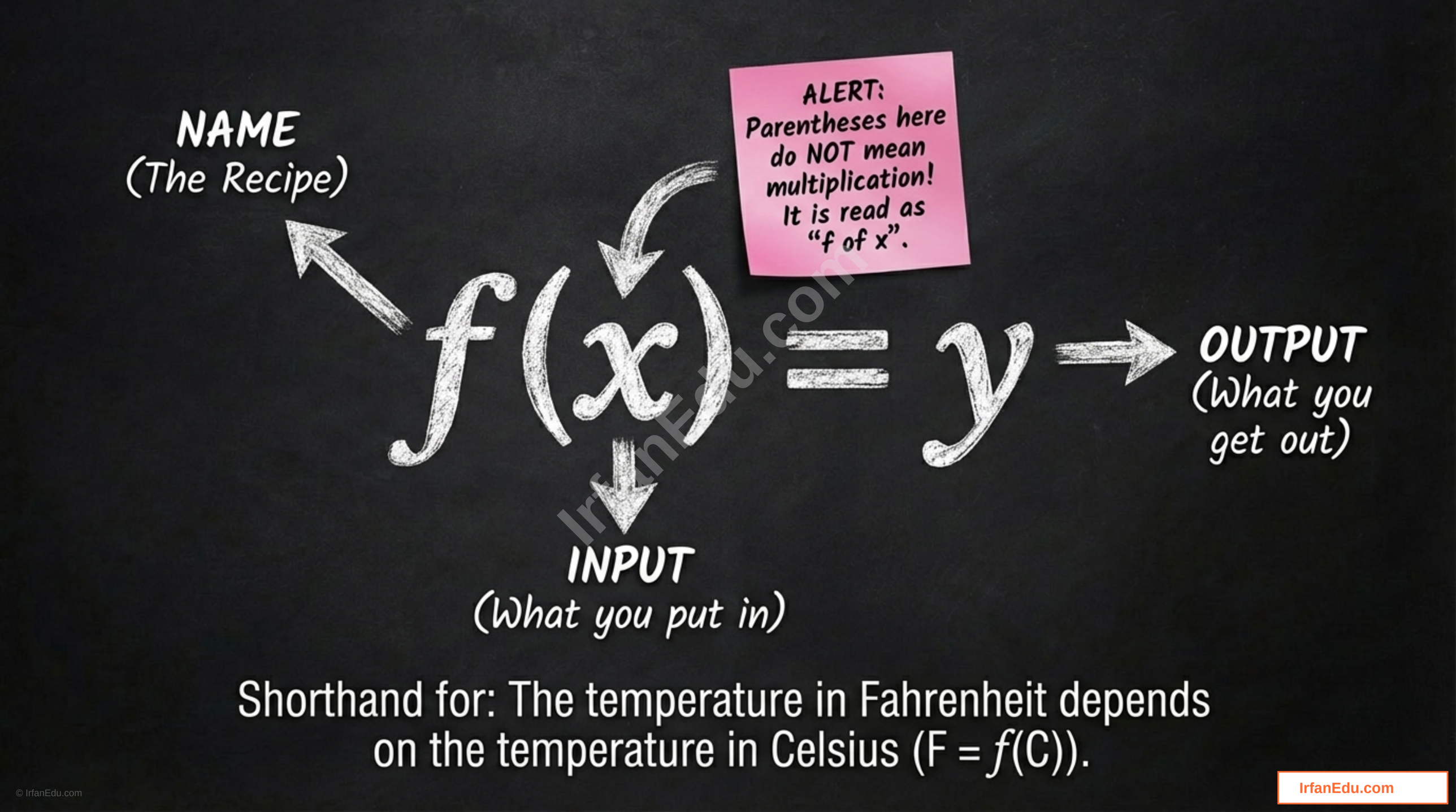
NOT A FUNCTION (Students → Colors)



BROKEN!
Alex can't pick two outputs. One input must have exactly ONE output.

THE QUICK CHECK METHOD





EVALUATING: THE MACHINE AT WORK

Given: $f(x) = 3x + 5$

Find $f(2)$:

$f(2) = 3(2) + 5$

$f(2) = 6 + 5$

$f(2) = 11$

Find $f(0)$:

$f(0) = 3(0) + 5$

$f(0) = 0 + 5$

$f(0) = 5$

Evaluating is just plugging in. We are asking the machine: "If I put in 2, what do I get?"

REPRESENTATION: TABLES

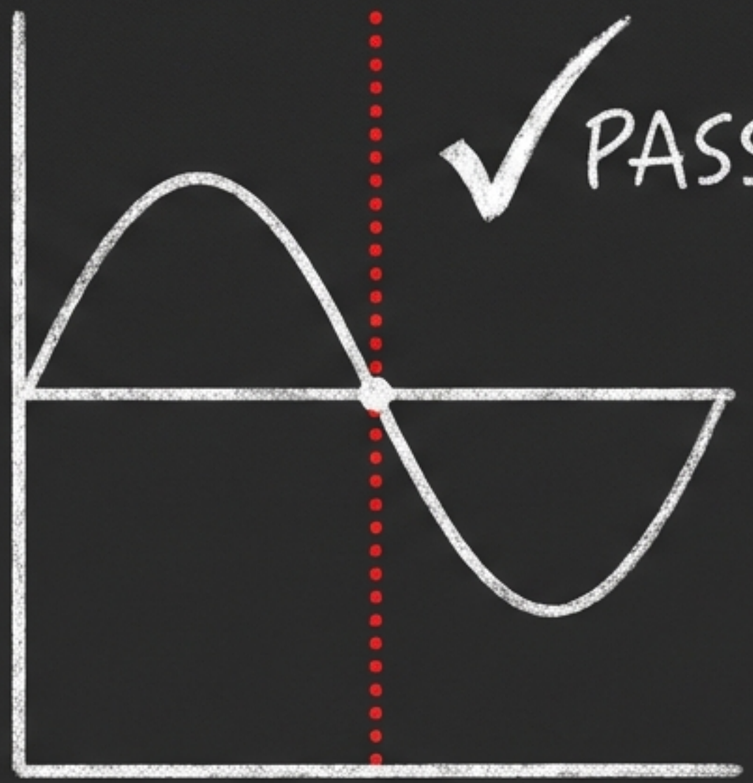
Instagram Growth

Month (m)	Followers $f(m)$
1	100
2	250
3	500
4	850
5	1200
6	1600

Scan the Input column.
If no numbers repeat, it is automatically a function.

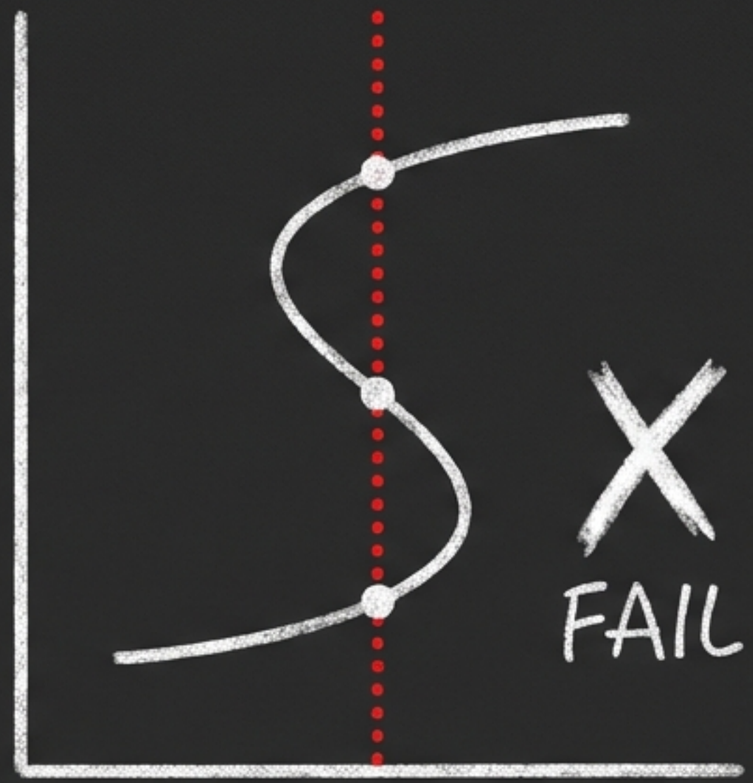
If an input DID repeat (like Month 3 appearing twice), check the output. If the output changes, it's broken!

THE VERTICAL LINE TEST



Graph A

The Mnemonic:
"V" for Vertical,
"F" for Function.
If the line hits more than once, it fails.



Graph B

SOLVING: WORKING BACKWARDS

Problem: Given $f(x) = 2x + 6$, solve for $f(x) = 14$.

Step 1: Set equal to 14 → $2x + 6 = 14$

Step 2: Subtract 6 ⟶ $2x = 8$

Step 3: Divide by 2 ⟶ $x = 4$

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Evaluating is finding the result. Solving is detective work: We have the result (14), we need to find the cause (x).

THE SPECIAL CLASS: ONE-TO-ONE

A function where each output corresponds to exactly one input.

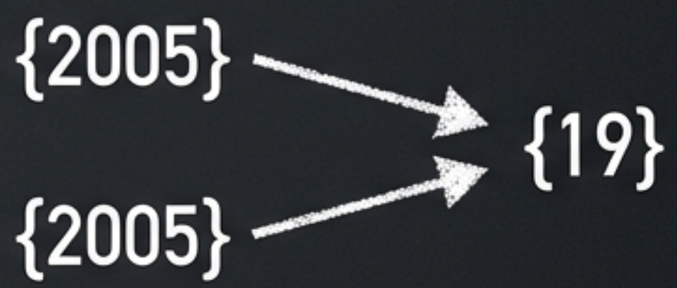
Student ID → Name
(One-to-One)

{1001} → {Emma}

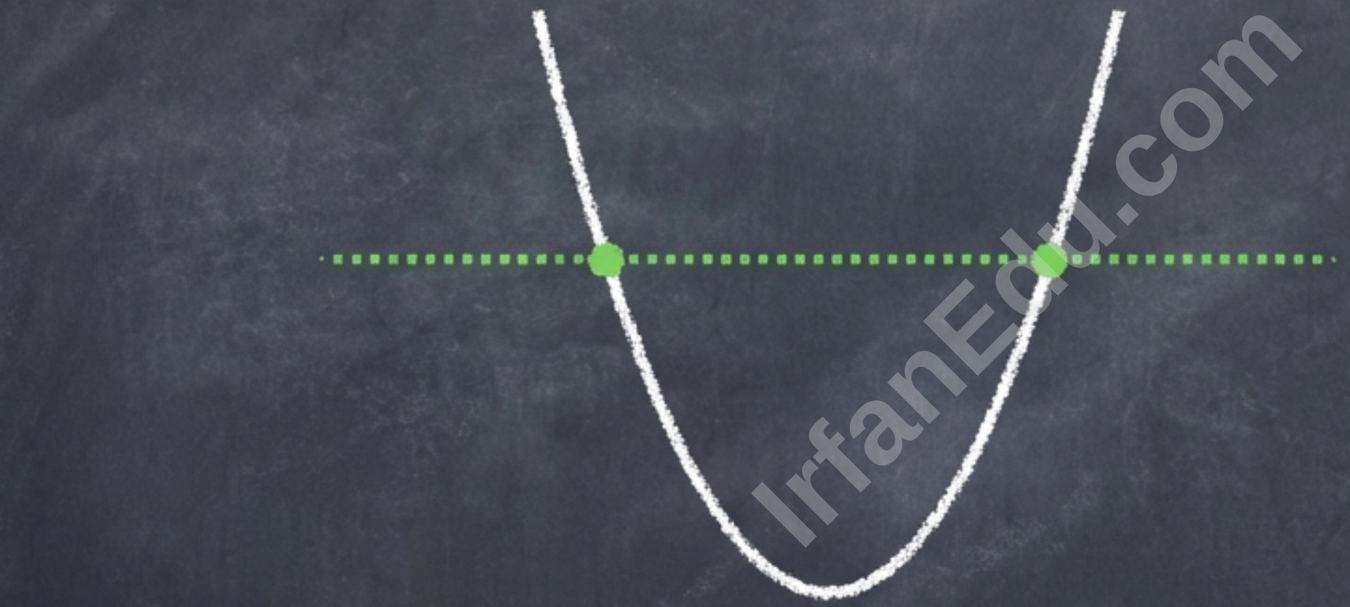
{1002} → {Liam}

One-to-One is the VIP section. No two inputs can share the same output.

Birth Year → Age
(Not One-to-One)



DIN Alternate Bold
THE HORIZONTAL LINE TEST



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Vertical Line =
Is it a Function?

Horizontal Line =
Is it Unique
(One-to-One)?

Fails Horizontal Test
This IS a function (passes vertical),
but NOT One-to-One.

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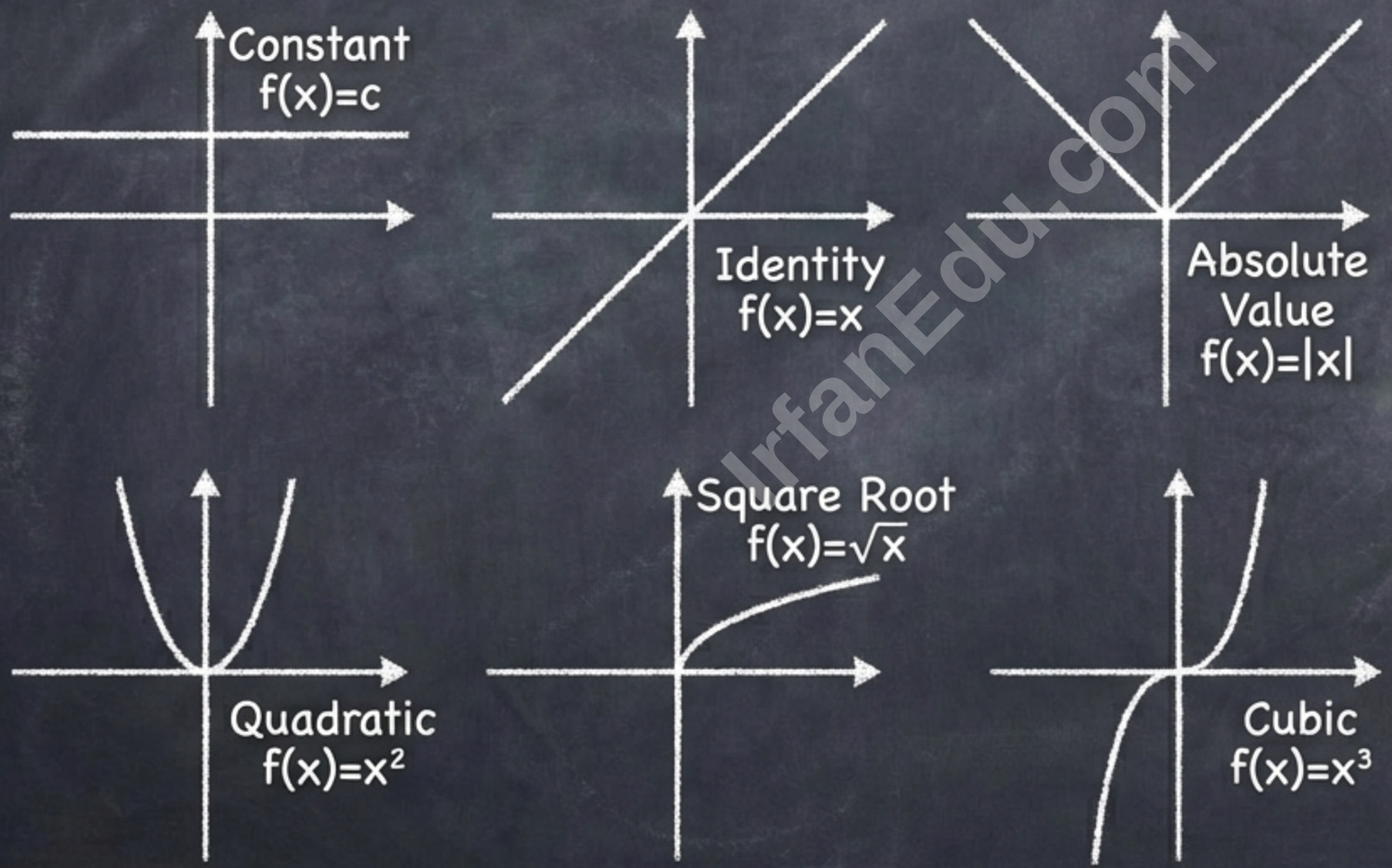
THE TOOLKIT: ESSENTIAL SHAPES



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These are the building blocks. Most complex math is just a variation of these 6 shapes.

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REAL WORLD APPLICATION: UBER

Formula: $C = f(m) = 3 + 1.50m$

(Cost = $3 base + $1.50 per mile)

Question: How much for a 10-mile ride?

Math: $f(10) = 3 + 1.50(10)$

$= 3 + 15$

$= \$18.00$

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Function notation in the real world looks like a receipt.

DIN Alternate Bold

REAL WORLD APPLICATION: FITNESS

Formula: C(m) = 100m
(Burning 100 calories per mile)

Problem: How far to burn 350 calories?

Math: 100m = 350 -> m = 3.5

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Here we solved for inputs. We knew the goal (350 cals) and calculated the required distance.

Answer: You need to run 3.5 miles.

CLASS DISMISSED: KEY TAKEAWAYS

- **Function:** Input → Exactly One Output.
- **Notation:** f(x) means ‘f of x’, not multiplication.
- **Evaluate:** Find Output (Plug it in).
- **Solve:** Find Input (Work backwards).
- **Vertical Test:** Checks if it’s a Function.
- **Horizontal Test:** Checks if it’s One-to-One.

Functions are just relationships. Inputs in, Outputs out. That's it!