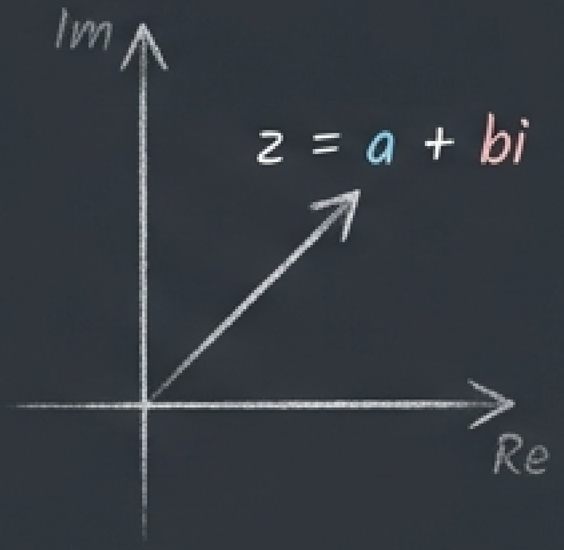
Arithmetic Operations with Complex Numbers

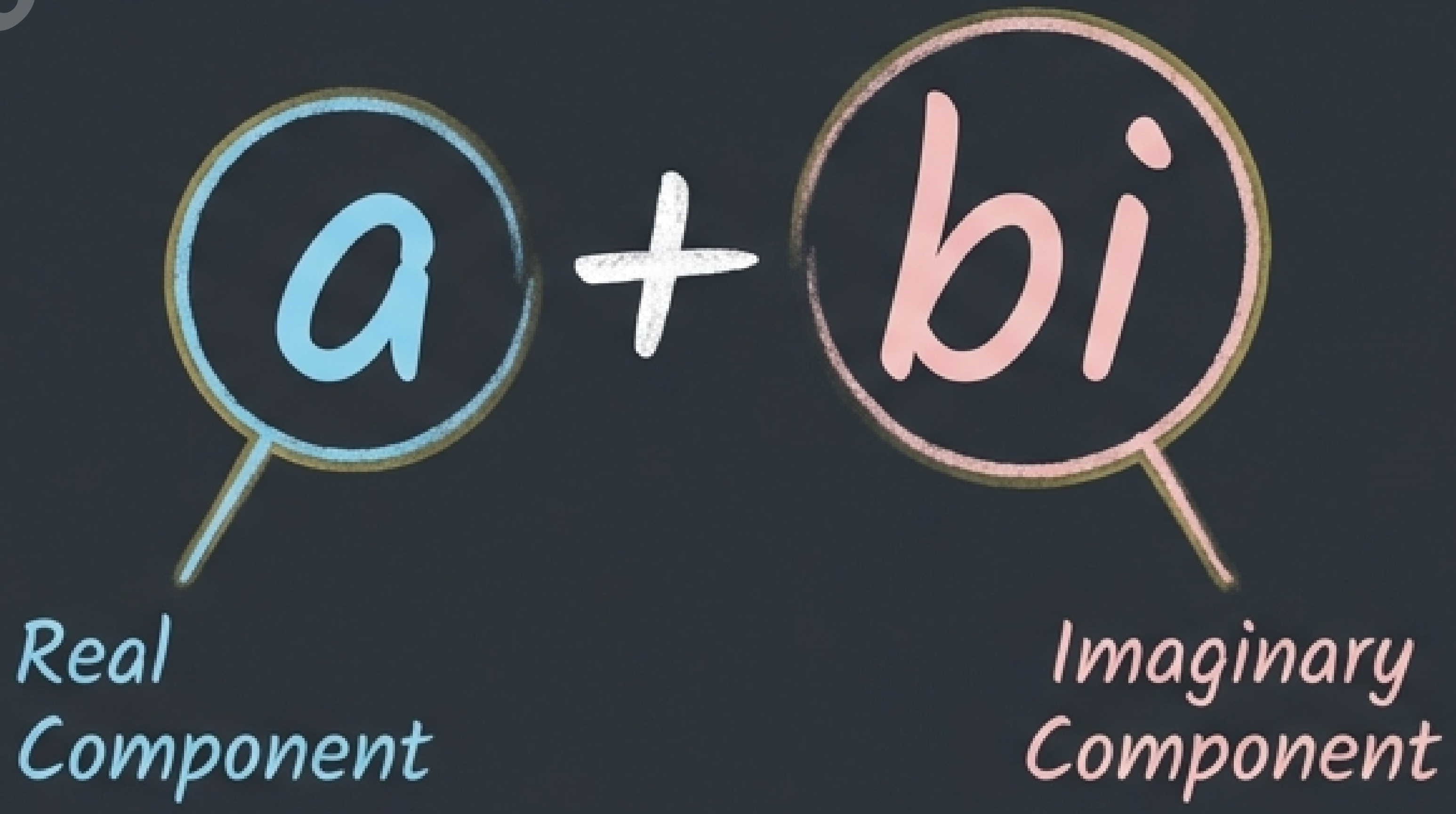
A Step-by-Step Guide to Mastery



Complex numbers follow the rules of algebra... with one twist.

We treat complex numbers like binomials (e.g., $x + y$).

We handle these entities just like everyday numbers—adding, subtracting, and multiplying—but we must always keep their identities distinct.



Addition requires organizing items into categories.

Straightforward approach:
Combine real components together.
Merge imaginary components separately.

$$(a+c)$$

$$(a+bi)+(c+di) = (a+c)+(b+d)i$$

$$(b+d)i$$

A practical example of adding components

Add (7 + 3i) and (5 - 2i)

Subtraction acts as addition with a distributed negative.

$$(a + bi) - (c + di)$$

$$(a - c) + (b - d)i$$

Teacher's Note: The distributive property is crucial here. Ensure the negative sign applies to both the real and imaginary parts of the second number.

Watching out for the double negative.

Calculate $(8 - 5i) - (3 + 2i)$

Step 1: Distribute	$8 - 5i - 3 - 2i$
Step 2: Group	$(8 - 3) + (-5i - 2i)$
Step 3: Solve	$5 - 7i$

Scaling a complex number by a real number

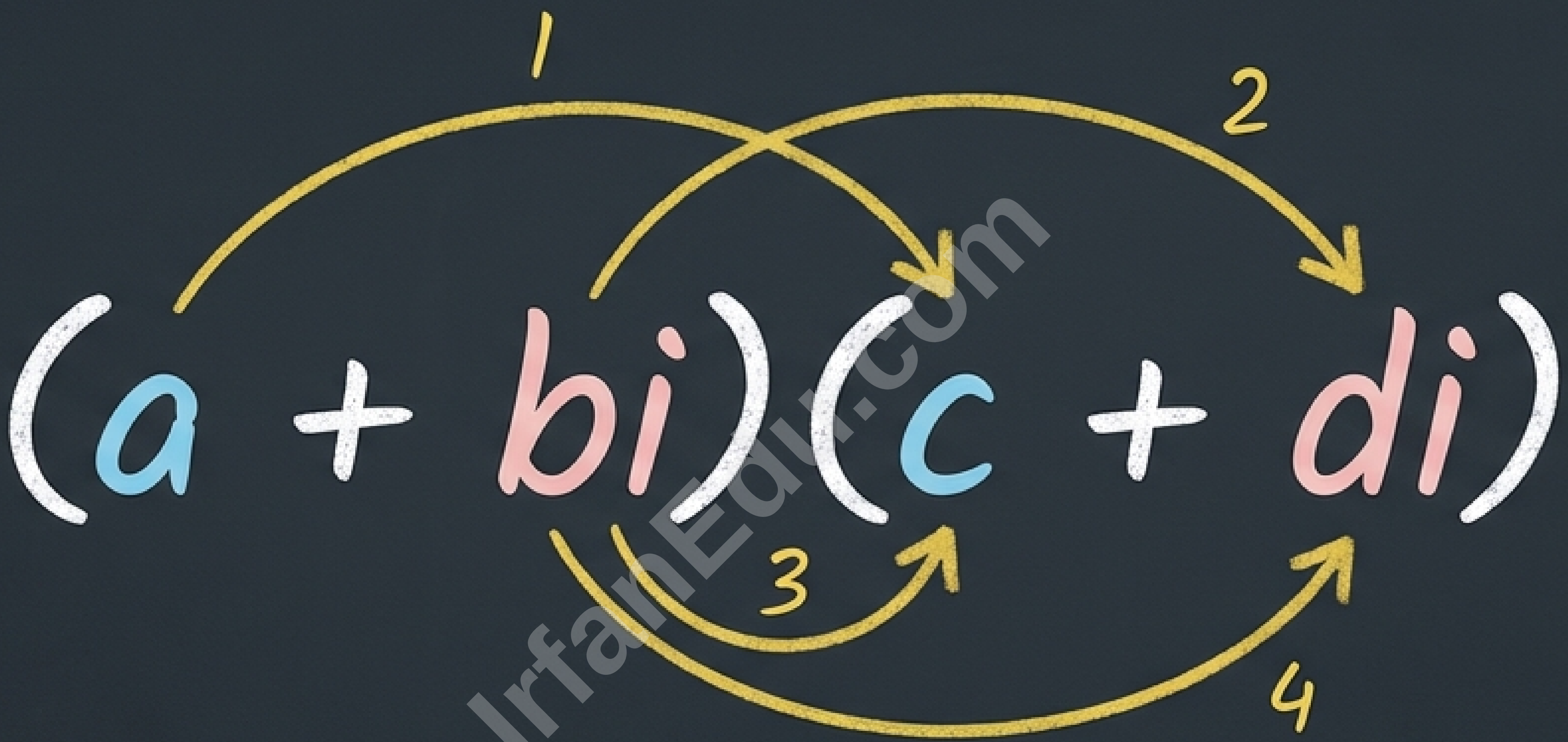
Distribute the real number to both the real and imaginary components.

$$5(3 + 4i)$$

$$5 \times 3 = 15 \qquad 5 \times 4i = 20i$$

$$15 + 20i$$

Multiplying two complex numbers using binomial distribution



$$(a + bi)(c + di)$$

$$ac + adi + bci + \underline{\underline{bdi^2}}$$

The defining property of imaginary numbers.

$$i^2 \rightarrow -1$$

This property transforms our calculations. While 'i' represents the imaginary unit, "i" squared becomes a Real number.

Complex Multiplication in Action (Part 1)

Multiply $(2 + 3i)(4 - i)$

$2 \cdot 4 = 8$	$2 \cdot (-i) = -2i$
$3i \cdot 4 = 12i$	$3i \cdot (-i) = -3i^2$

$$8 - 2i + 12i - 3i^2$$

Resolving the imaginary square.

$$8 - 2i + 12i - 3i^2$$

$$-3(-1) = +3$$

$$8 - 2i + 12i + 3$$

$$(8 + 3) + (-2i + 12i)$$

$$11 + 10i$$

A visual guide to operations.

Add/Subtract	Combine like terms separately.	$(\text{Real} \pm \text{Real}) + (\text{Imag} \pm \text{Imag})i$
Multiplication	Use distributive property.	F O I L: $(a + bi)(c + di)$
The Golden Rule	Remember the substitution.	$i^2 \rightarrow -1$

Practice problems for mastery.

Apply the principles of grouping and distribution to solve these exercises.

The Questions

1. Add $(6 - 2i) + (4 + 7i)$
2. Subtract $(9 + 3i) - (5 - 4i)$
3. Multiply $(1 + 2i)(3 - i)$

Solutions breakdown.

1) $(6 + 4) + (-2i + 7i) = \boxed{10 + 5i}$

2) $(9 - 5) + (3i + 4i) = \boxed{4 + 7i}$

3) $3 - i + 6i - 2i^2 \rightarrow 3 + 5i - \underline{2(-1)} \rightarrow 3 + 5i + 2 = \boxed{5 + 5i}$

Key takeaways for success.

- Mirror Real Arithmetic: Operations follow logical patterns.
- Segregate: Always combine Real with Real and Imaginary with Imaginary.
- The Twist: In multiplication, carefully substitute $i^2 = -1$.
- Standard Form: Always express final answers as $a + bi$.

Complex numbers aren't complex once you grasp these fundamental principles.