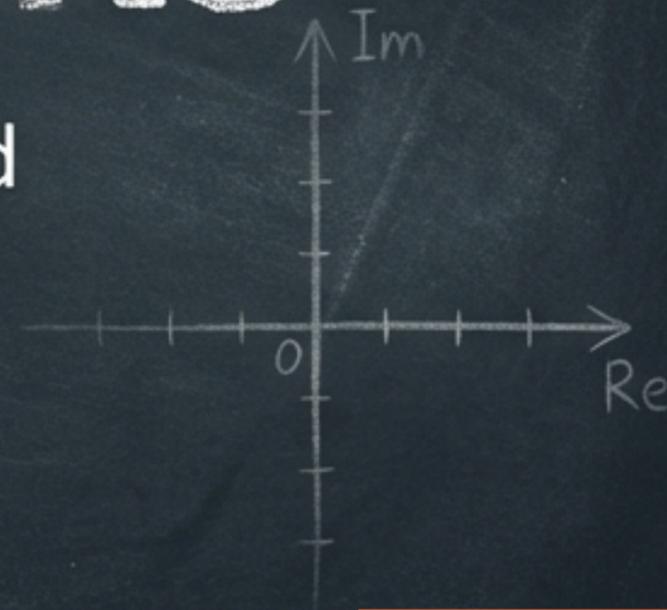


MASTERING OPERATIONS ON COMPLEX NUMBERS

A Step-by-Step Guide to the Imaginary World



Real Part
Caveat Brush

$a + bi$

Imaginary
Part

The Definition: $i = \sqrt{-1}$
in DIN Rounded

The Golden Rule: $i^2 = -1$
in DIN Rounded

*Think of these as
coordinates on a map.*

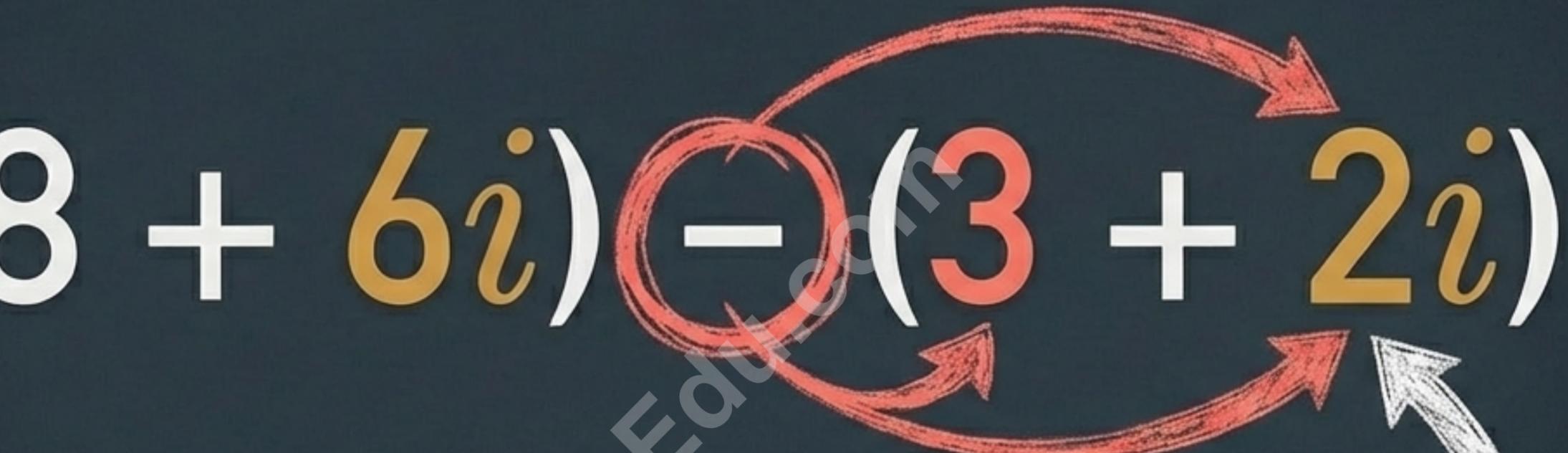
ADDITION: COMBINING LIKE TERMS

$$(5 + 3i) + (2 + 7i)$$

*Real adds to Real.
Imaginary adds to Imaginary.*

$$7 + 10i$$

SUBTRACTION: THE DISTRIBUTION TRAP

$$(8 + 6i) - (3 + 2i)$$


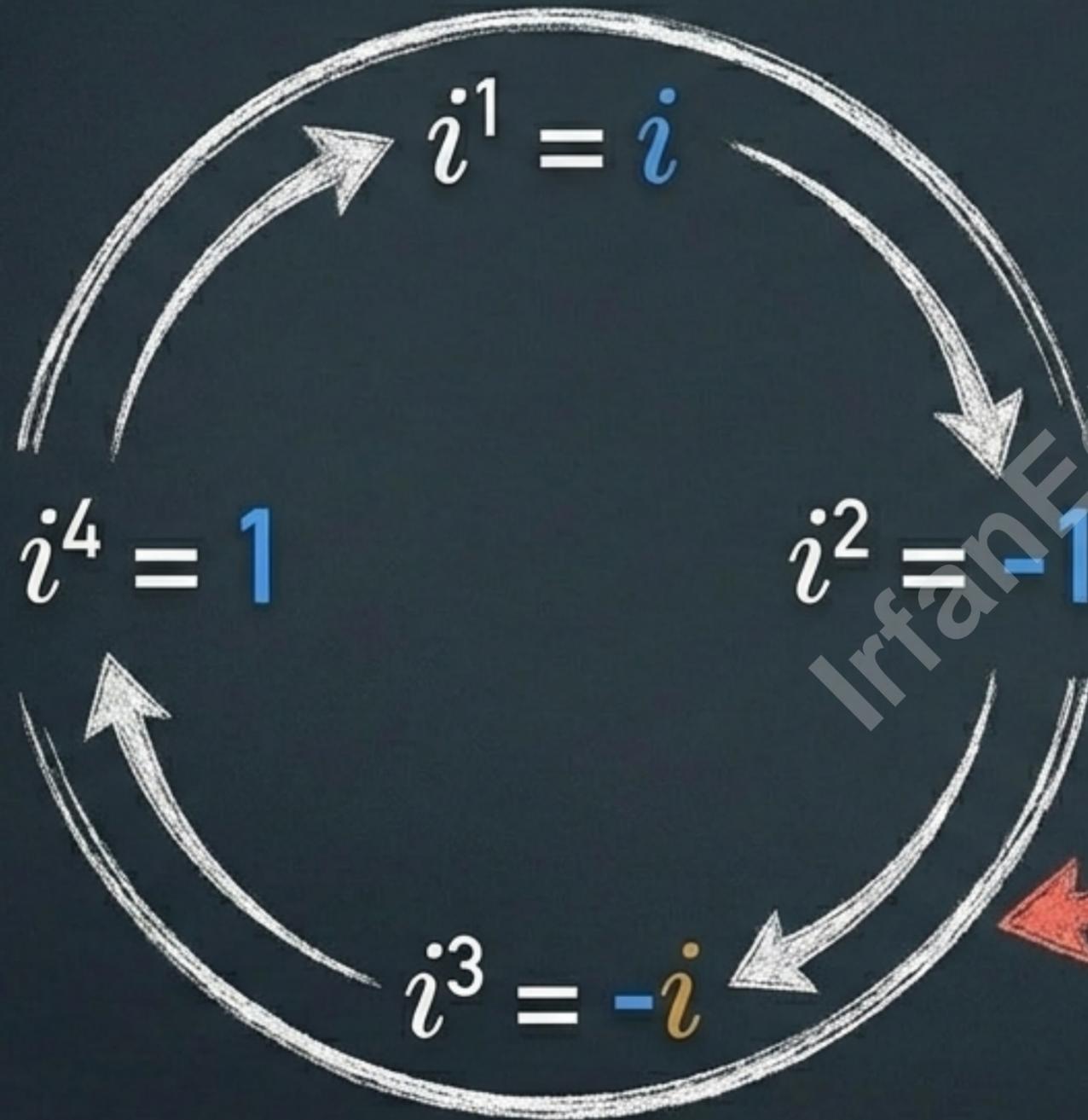
$$\text{Real: } 8 - 3 = 5$$

$$\text{Imaginary: } 6i - 2i = 4i$$

$$5 + 4i$$

*Don't forget to
distribute the
negative here!*

THE CYCLE OF POWERS



The Quarter Trick:
Divide the exponent by 4
and check the remainder.

Solve i^{47}

$$47 \div 4 = 11 \text{ R } 3$$

$$i^{47} = -i$$

MULTIPLICATION: THE FOIL METHOD

First: 12

Outer: 15i

Inner: 8i

Last: 10i²

$$(3 + 2i)(4 + 5i)$$

Wait for the twist...

Tungsten

MULTIPLICATION: THE TRANSFORMATION

$$12 + 15i + 8i + \underline{10i^2} \rightarrow 10(-1) \rightarrow -10$$

The i^2 turns the last term from imaginary back to real.

Real: $12 - 10 = 2$ Imaginary: $15i + 8i = 23i$

$$2 + 23i$$

THE CHALLENGE OF DIVISION

$$6 + 8i$$

$$2 + 3i$$

Rule: No imaginary numbers allowed in the denominator.

The Solution: The Conjugate

$$2 + 3i \rightarrow 2 - 3i$$

Change *ONLY* the middle sign.

Tungsten

DIVISION: SETTING THE TRAP

$$\frac{6 + 8i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} \quad \left. \vphantom{\frac{6 + 8i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}} \right\} \text{Value} = 1$$

Caveat Brush
We aren't changing the
value, just the format.

STEP 1: THE NUMERATOR

$$\frac{(6 + 8i)(2 - 3i)}{(6 + 8i)(2 - 3i)}$$

Focus on the top bars. The denominator is for later.

$$12 - 18i + 16i - 24i^2$$

becomes +24

$$\text{Real: } 12 + 24 = 36$$

$$\text{Imaginary: } -18i + 16i = -2i$$

$$\text{New Numerator} = 36 - 2i$$

STEP 2: THE DENOMINATOR

$$\frac{(6 + 8i)(2 - 3i)}{(2 + 3i)(2 - 3i)}$$

*Difference of Squares
Shortcut $(a^2 + b^2)$*

$$2^2 + 3^2$$

$$4 + 9$$

*Notice: The "i" has
completely
disappeared!*

$$\text{New Denominator} = 13$$

DIVISION: THE FINAL ANSWER

$$\frac{(36 - 2i)}{13}$$

$$\frac{36}{13}$$

(Real Part)



$$-\frac{2}{13}i$$

(Imaginary Part)

$$36/13 - 2/13 i$$



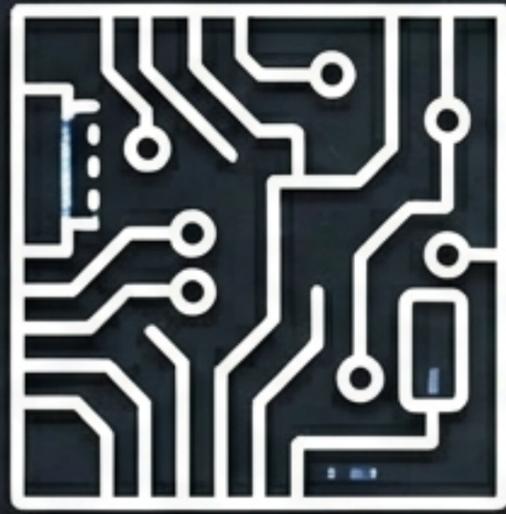
Always separate the final answer into a + bi form.

SAFETY CHECKS & PITFALLS

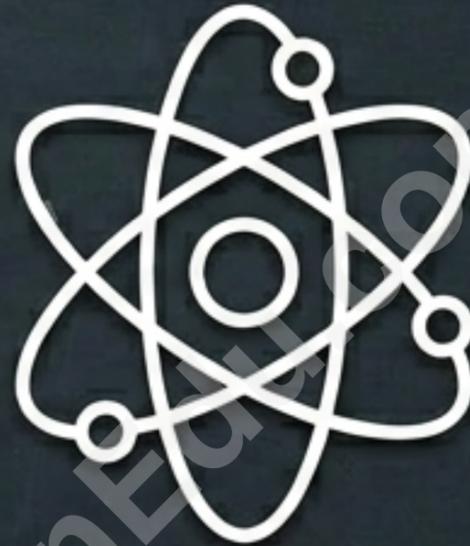
- ✓ The i^2 Switch: Did you replace every i^2 with -1 ?
- ✓ The Distribution: Did you distribute the negative sign to BOTH parts?
- ✓ The Conjugate: Did you change ONLY the imaginary sign?
- ✓ Standard Form: Is the result separated into $a + bi$?



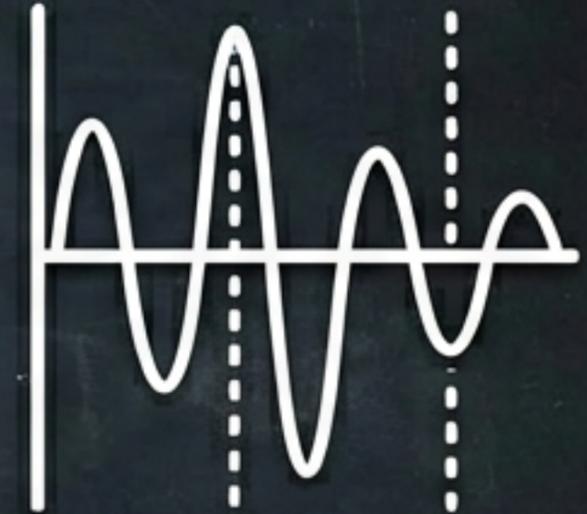
REAL WORLD APPLICATIONS



Electrical Engineering
(AC Circuits)



Quantum
Mechanics



Signal
Processing

Complex numbers are the mathematical backbone of modern technology.

SUMMARY CHEAT SHEET

ADD/SUBTRACT

Combine Like Terms. Distribute negatives.

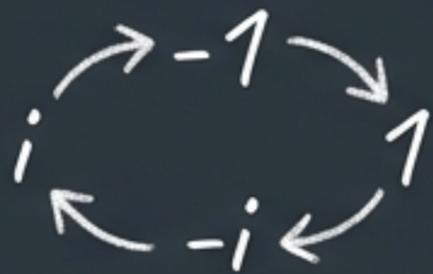
$$\underline{(a+c)} + \underline{(b+d)i}$$

MULTIPLY

FOIL Method. Replace i^2 with -1 .

$$\underline{ac} - \underline{bd} + \underline{(ad+bc)i}$$

POWERS



Divide exponent by 4.

DIVIDE

Multiply top and bottom by Conjugate.

$$\underline{(a+bi)}(\underline{a-bi}) = \underline{a^2 + b^2}$$

You are now ready to master the complex.