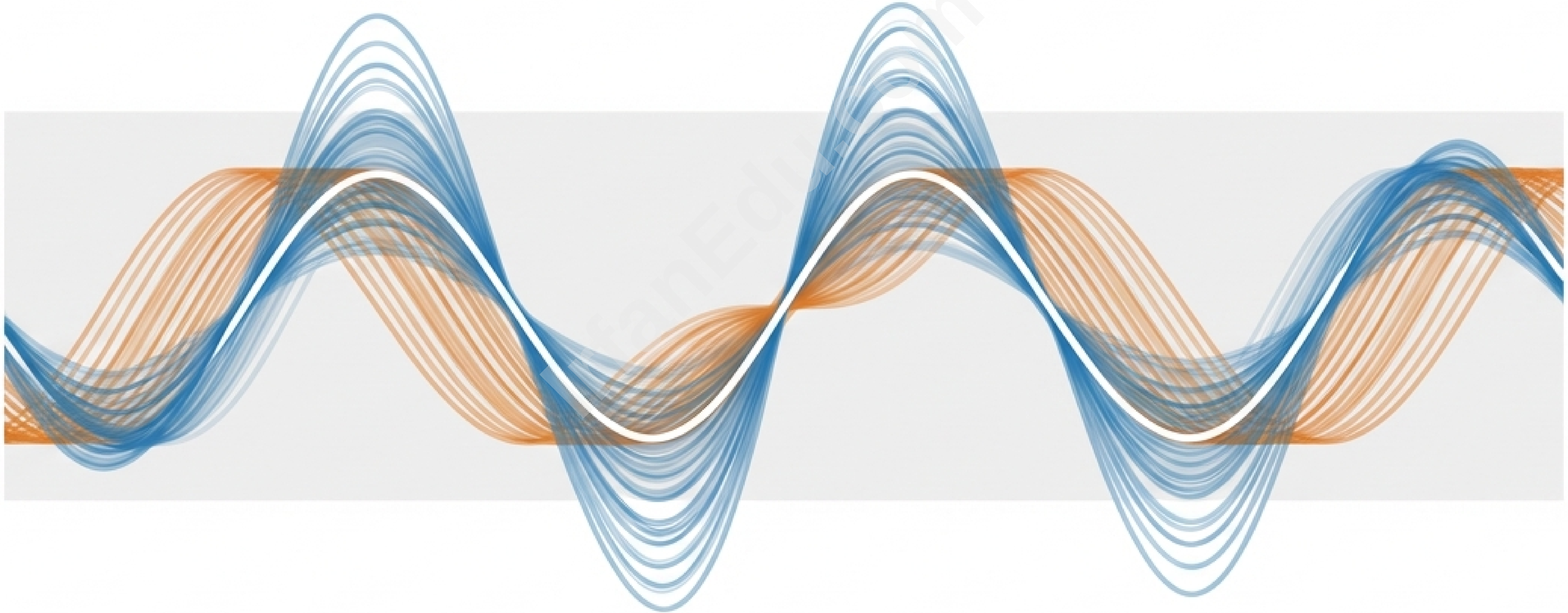
Mastering Graph Transformations

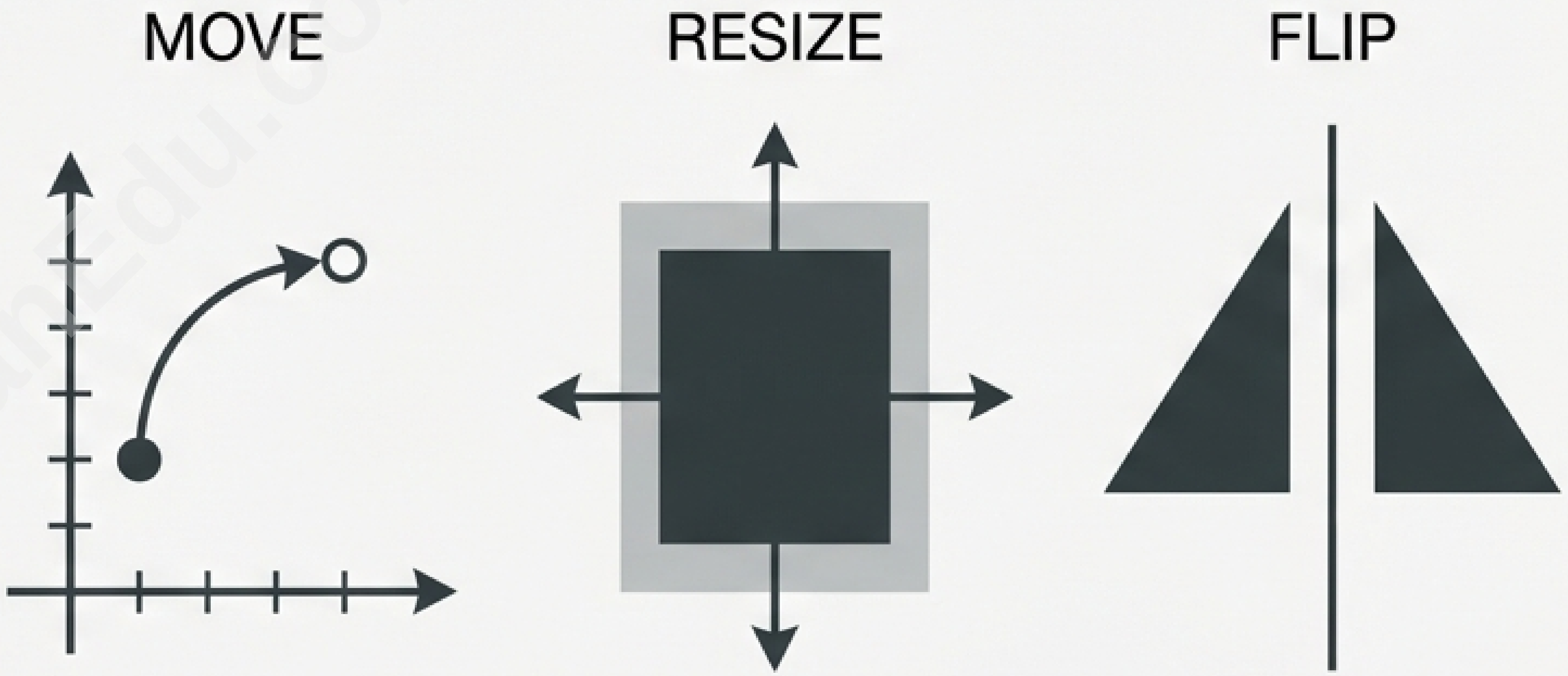
A Visual Guide to Manipulating Functions



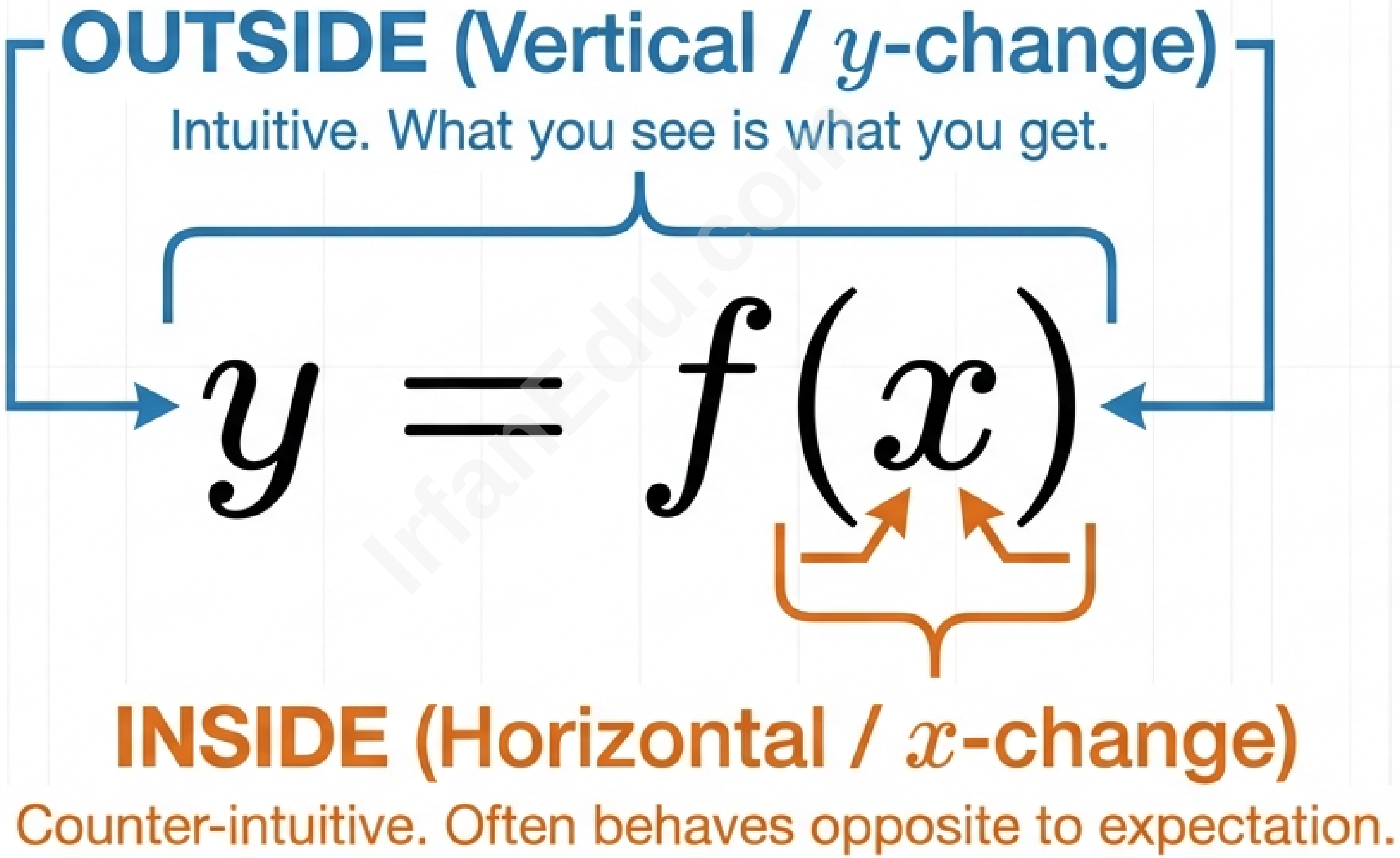
Reshaping Graphs Without Breaking the Rules

Graph transformations modify the position, shape, or orientation of a function without changing its fundamental characteristics. Think of them as a set of instructions—like -ike code—that tell a graph how to move.

The Insight: Mastering these patterns allows you to predict visual changes just by looking at the equation.



The Golden Rule: Location Determines Effect



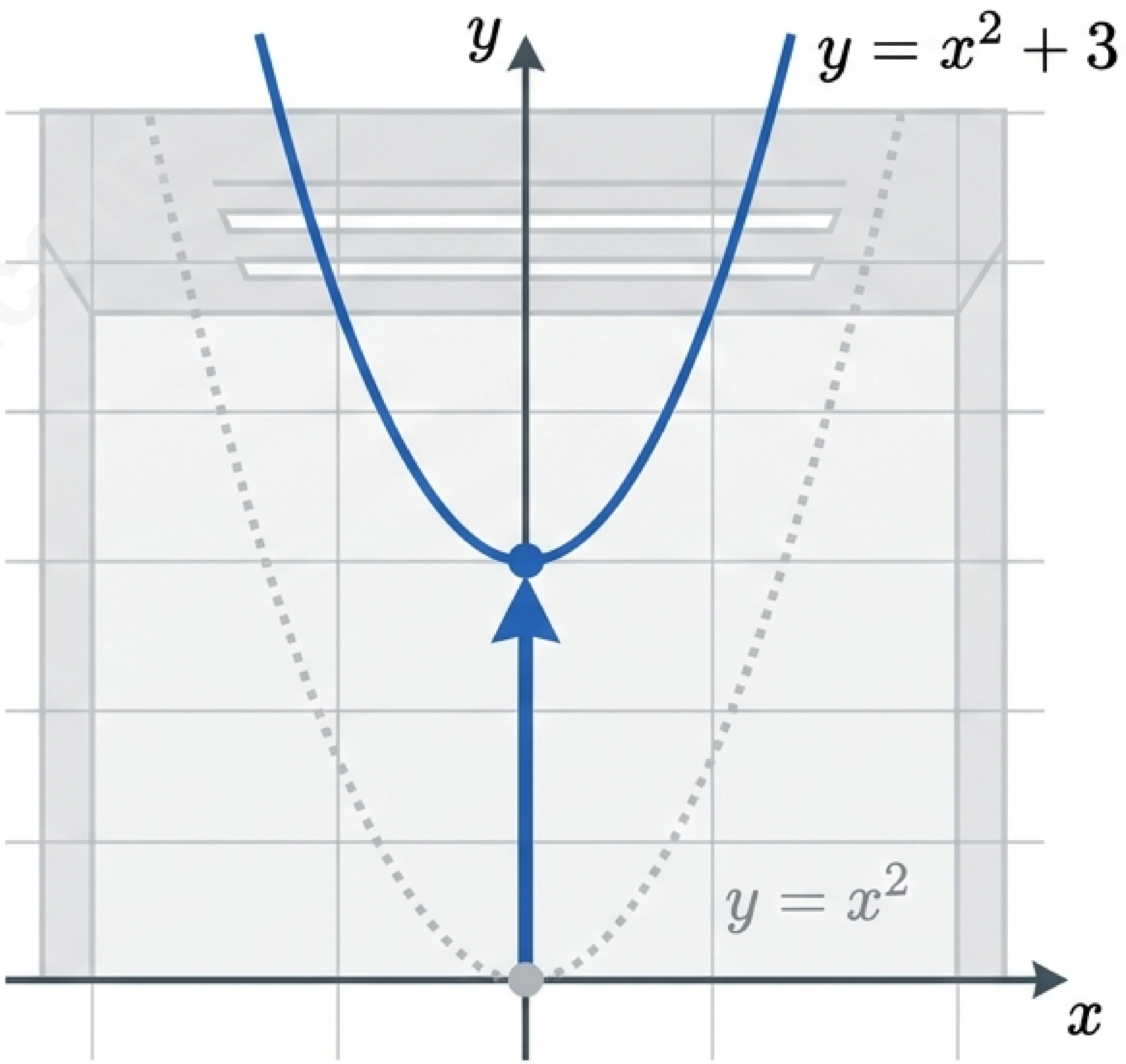
Vertical Shifts: The Elevator

$$g(x) = f(x) + k$$

The Rule:
If k is positive (+), shift **UP**.
If k is negative (-), shift **DOWN**.

Why?
Since k is added to the final output, every point moves vertically by the exact same amount.

Example:
$x^2 + 3$ shifts the parabola up 3 units.



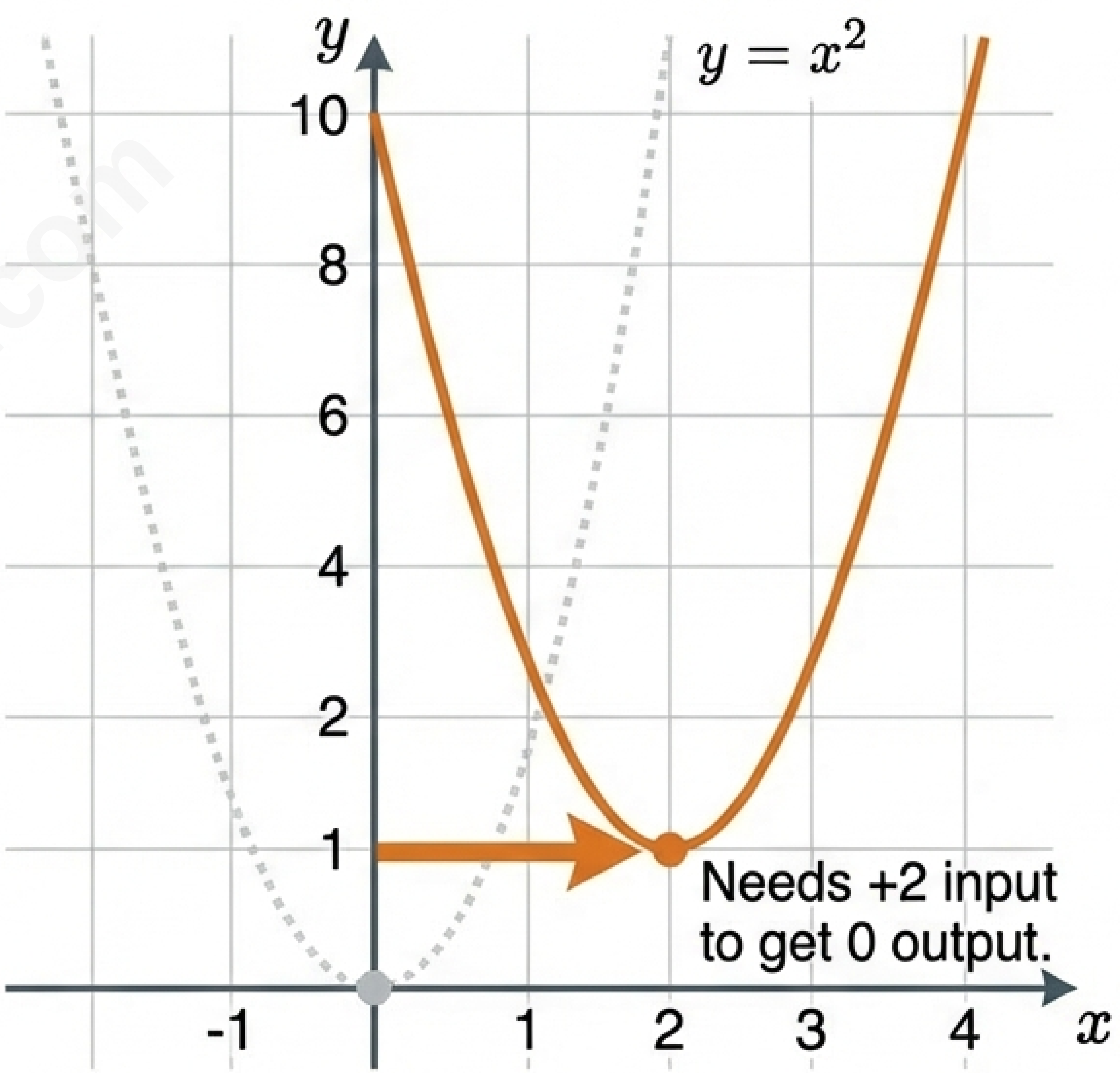
Horizontal Shifts: The Time Traveler

$$g(x) = f(x - h)$$

The Student Trap:
Why does $(x - 2)$ move Right?

The Logic:
To get the same result ($y = 0$), the input x must be positive 2. The function is “waiting” for a larger number to equalize the equation.

The Rule:
$(x - h)$ moves **RIGHT** (Positive direction).
$(x + h)$ moves **LEFT** (Negative direction).



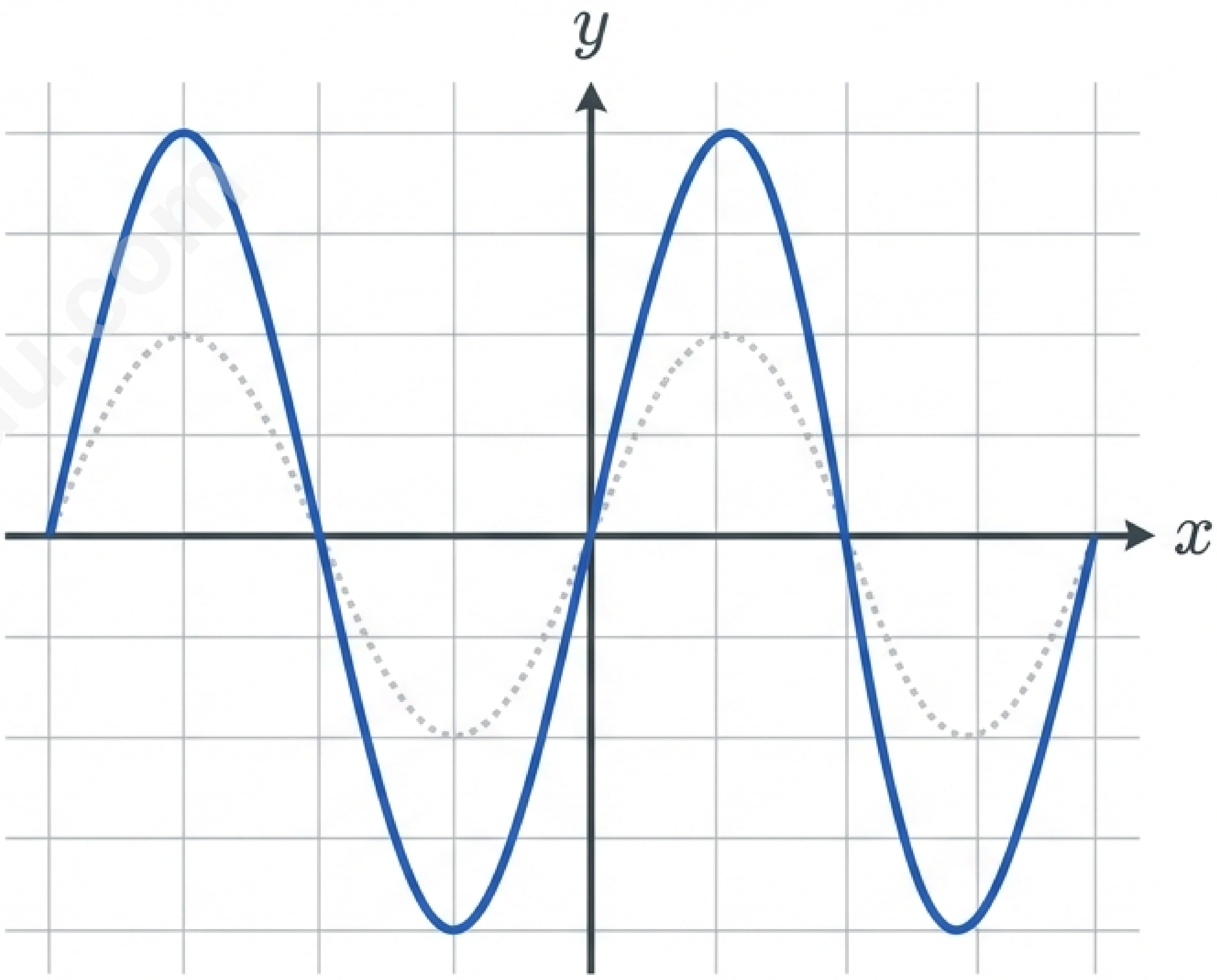
Vertical Stretches & Compressions

$$g(x) = a \cdot f(x)$$

Logic:
You are multiplying the output y.

Stretch ($|a| > 1$):
Graph gets taller/narrower.
Points move away from x-axis.

Compress ($0 < |a| < 1$):
Graph gets shorter/wider.
Points move closer to x-axis.



Multiplying by 2 pulls the graph vertically.

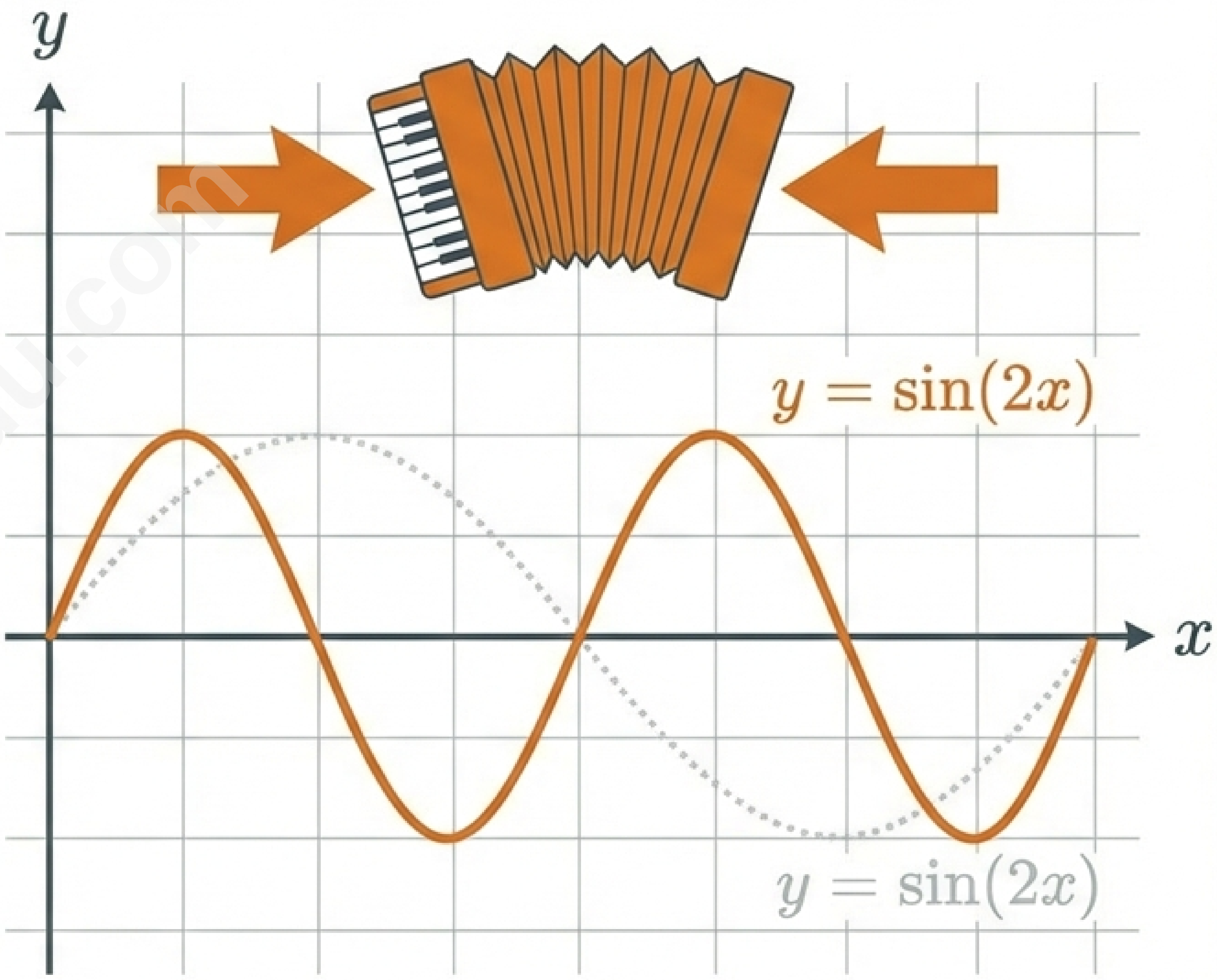
Horizontal Stretches: The Accordion

$$g(x) = f(bx)$$

Counter-Intuitive Rule:
Inside changes define the *speed*.

$b > 1$ (e.g., $2x$):
Graph **Compresses**. It happens faster, so it takes up less space.

$0 < b < 1$ (e.g., $0.5x$):
Graph **Stretches**. It happens slower, taking up more space.

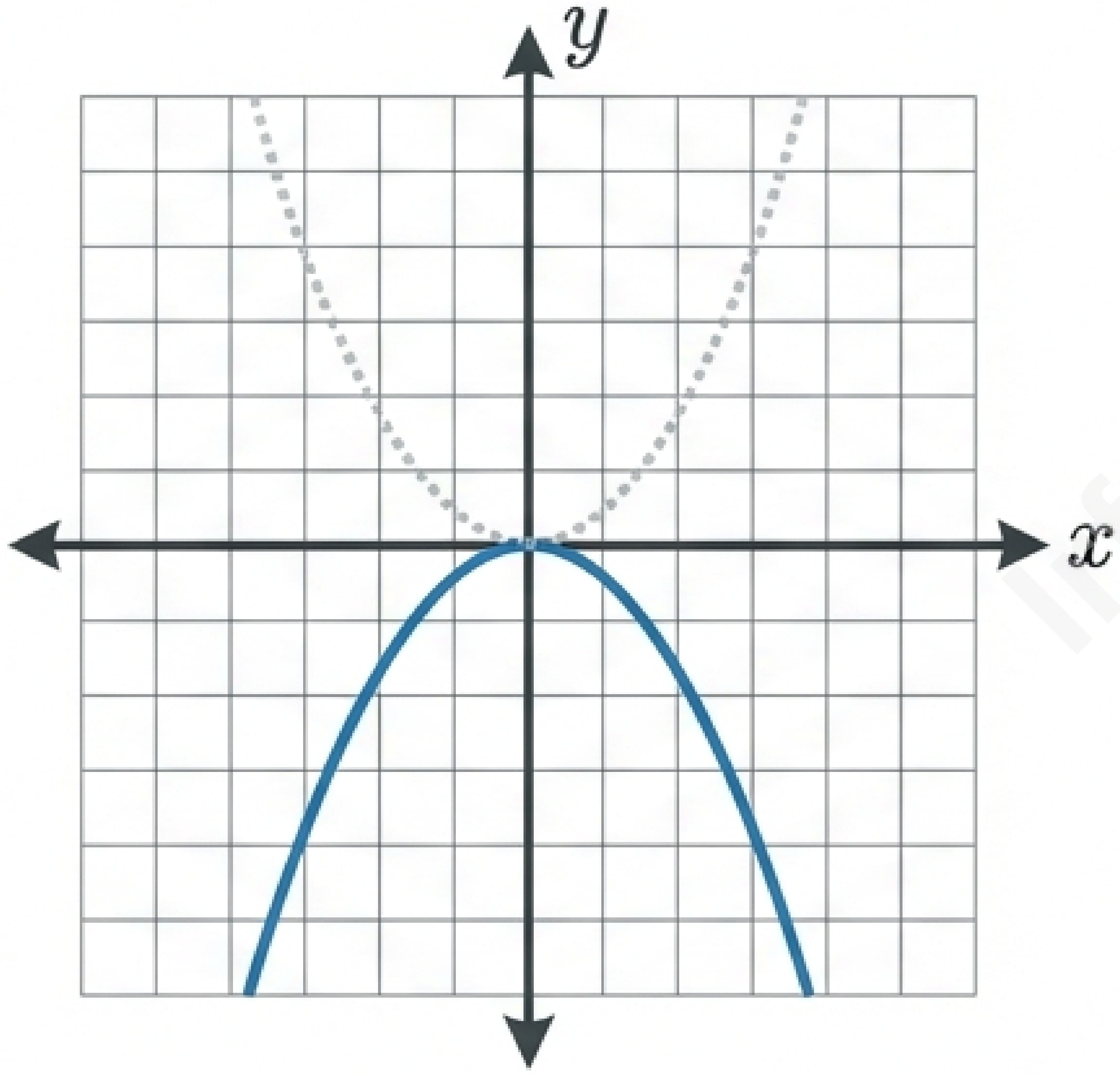


$f(2x)$ implies happening twice as fast → half the space.

Reflections: The Mirror Effect

Vertical Reflection (Over x-axis)

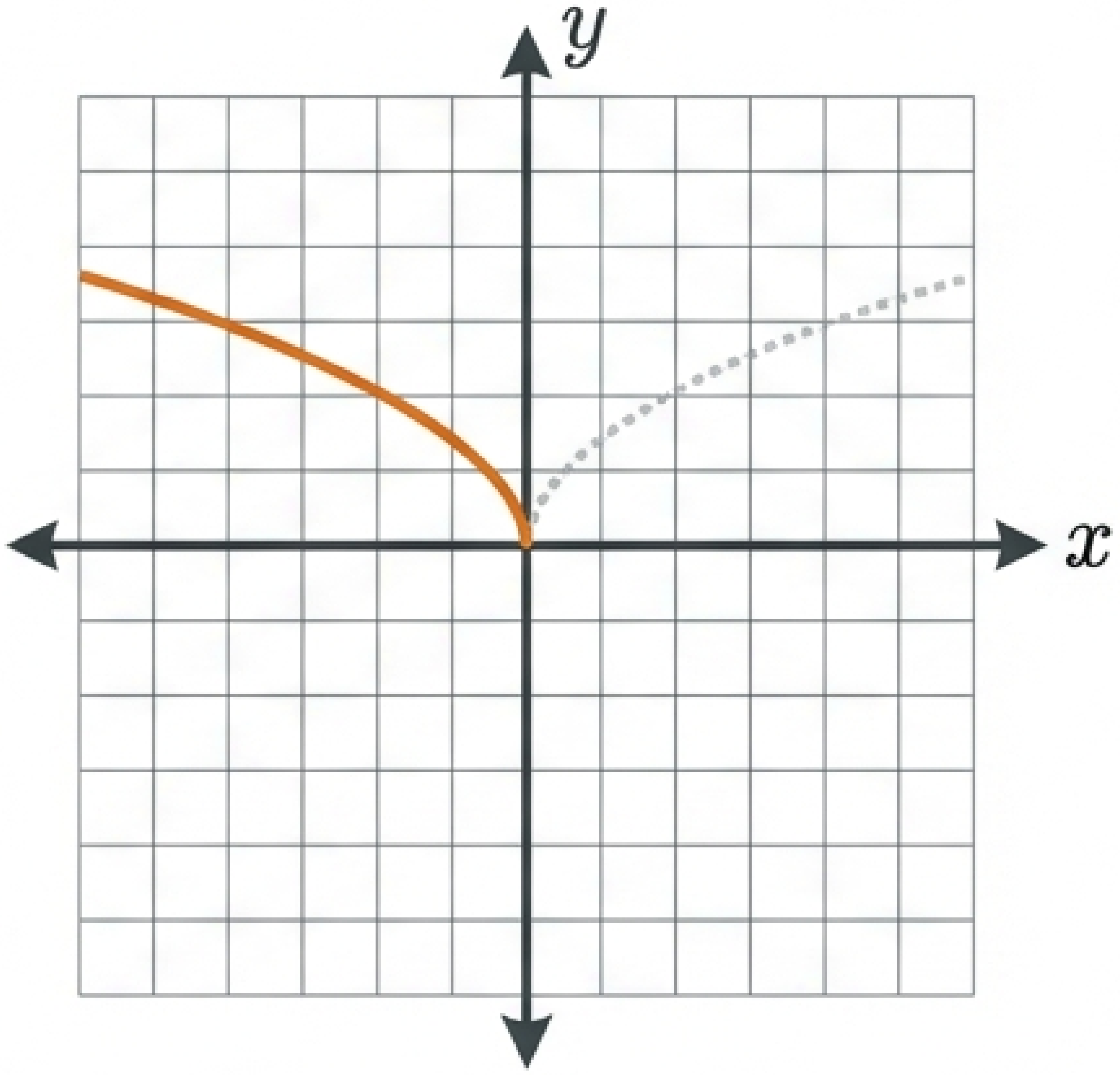
$$g(x) = -f(x)$$



Negative is OUTSIDE. y becomes $-y$.

Horizontal Reflection (Over y-axis)

$$g(x) = f(-x)$$



Negative is INSIDE. x becomes $-x$.

The Roadmap: Order of Operations

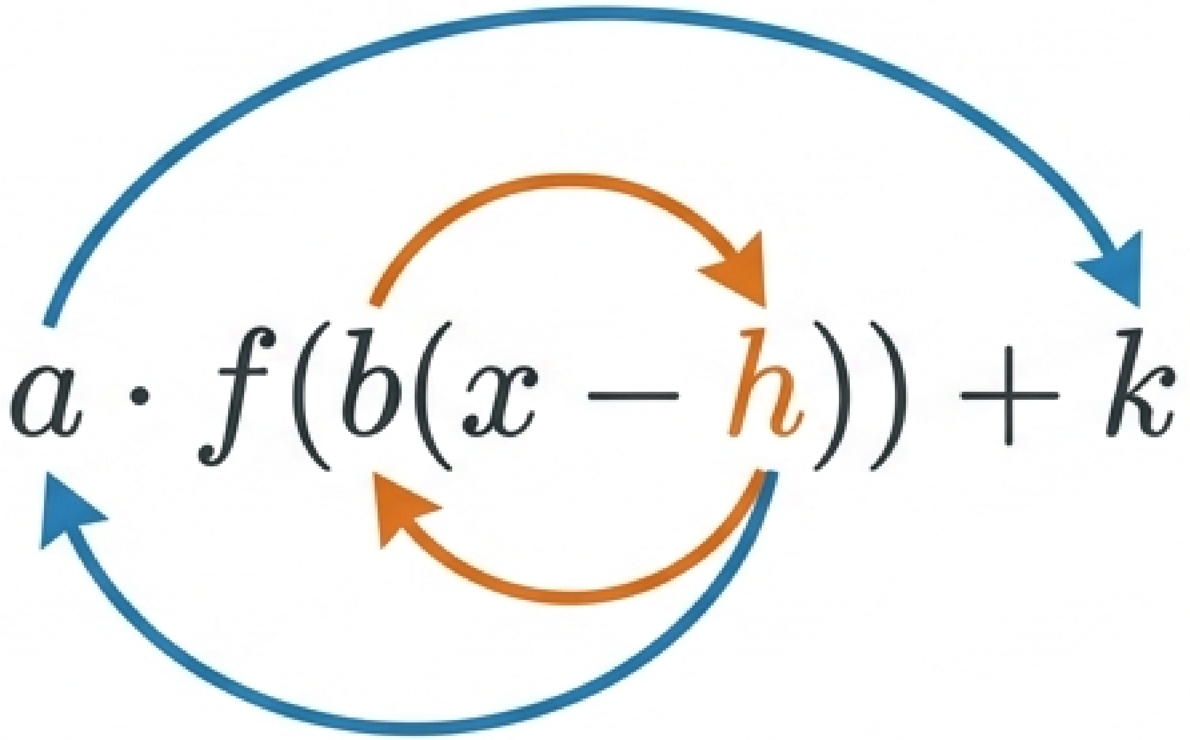
Work from the Inside Out

1. Horizontal Shifts
(Inside: Add/Subtract)

2. Horizontal Stretch/Reflect
(Inside: Multiply/Negate)

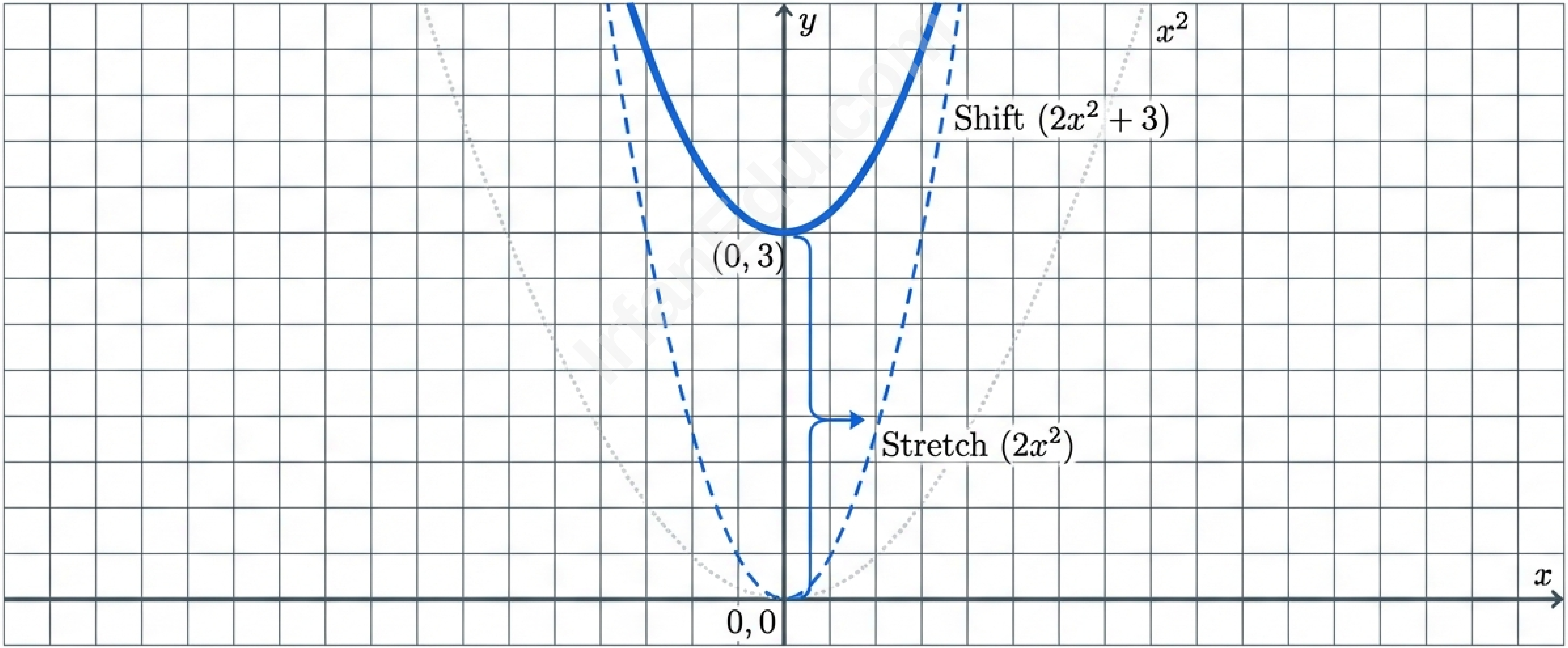
3. Vertical Stretch/Reflect
(Outside: Multiply/Negate)

4. Vertical Shifts
(Outside: Add/Subtract)



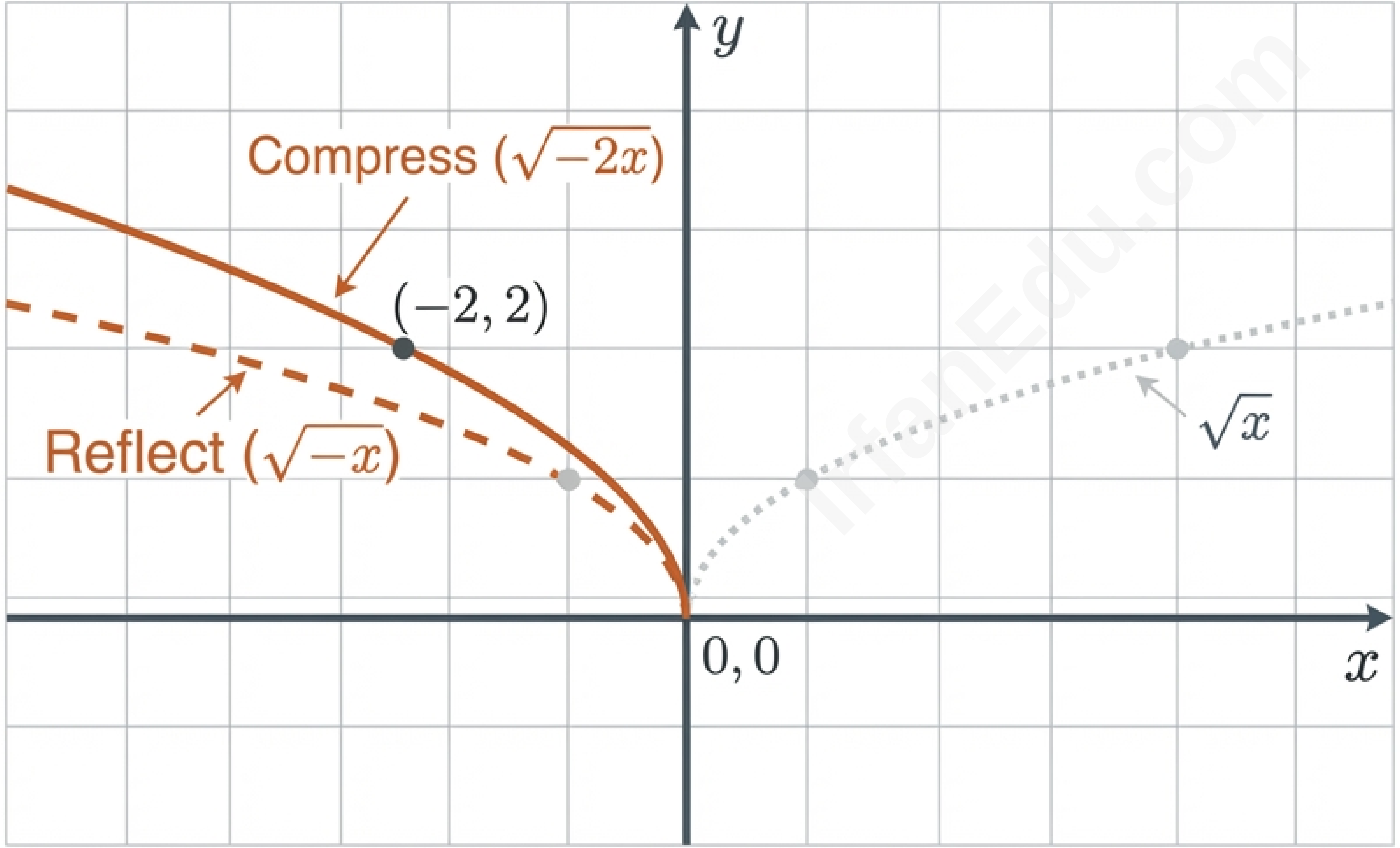
Example 1: Vertical Combination

Transform $f(x) = x^2$ into $g(x) = 2x^2 + 3$



Example 2: Horizontal Compression & Reflection

Transform $f(x) = \sqrt{x}$ into $g(x) = \sqrt{-2x}$

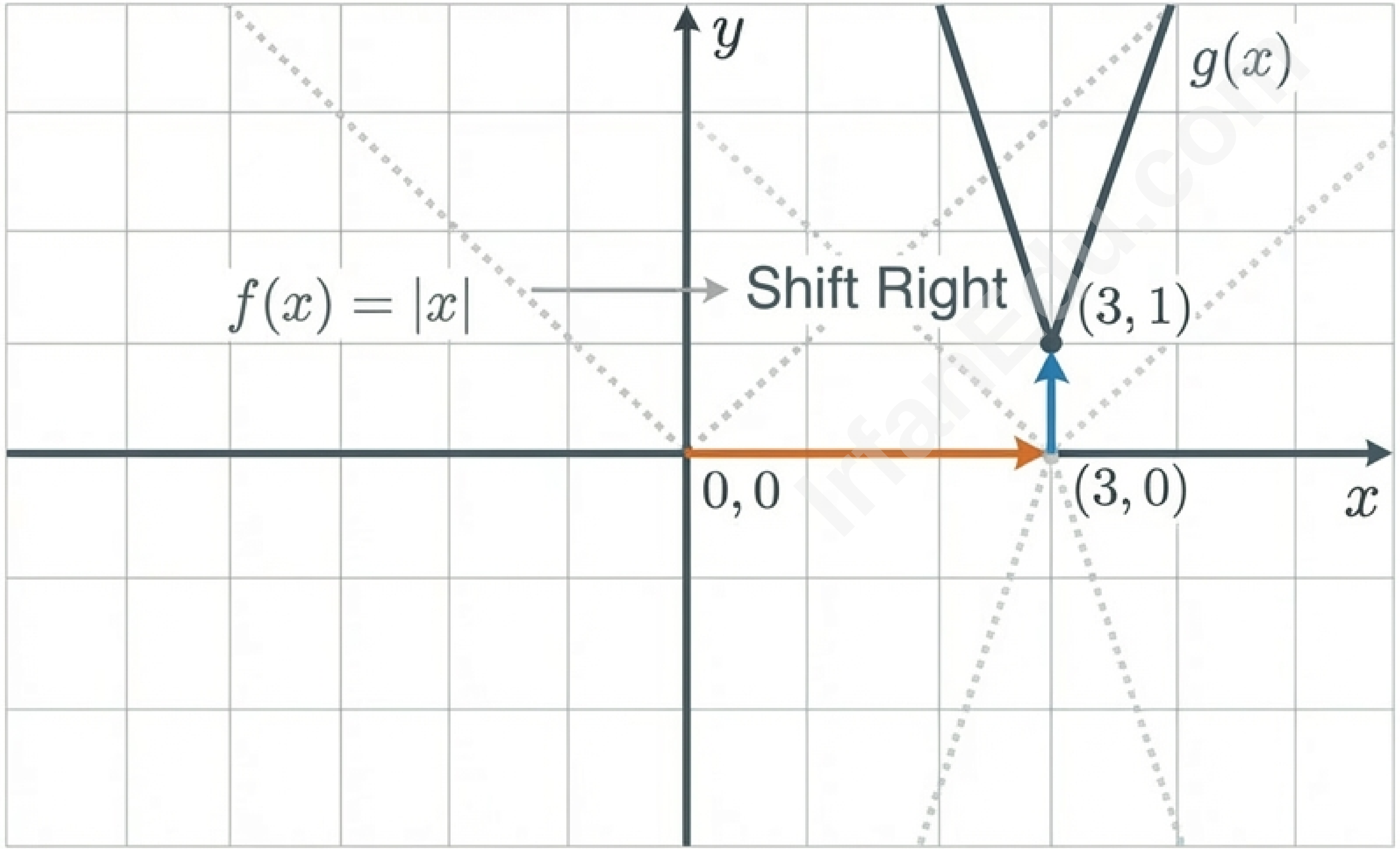


Analysis:

1. **Reflection**: Negative inside flip over y-axis.
2. **Compression**: Coefficient 2 implies half the width.

Example 3: The Master Class

Transform $f(x) = |x|$ into $g(x) = -2|x - 3| + 1$



Analysis:

1. **Right 3** $(x - 3)$
2. **Vertical Stretch** (2) -
   Vertical Stretch (2)
3. **Vertical Reflect** $(-\$)$
4. **Up 1** $(+1)$

Troubleshooting Common Mistakes

Direction Confusion

Thinking $(x - 2)$ moves Left.

Correction (Bold):

Remember: Minus moves **Right**. The function is “waiting” for a larger number.

The Stretch Trap

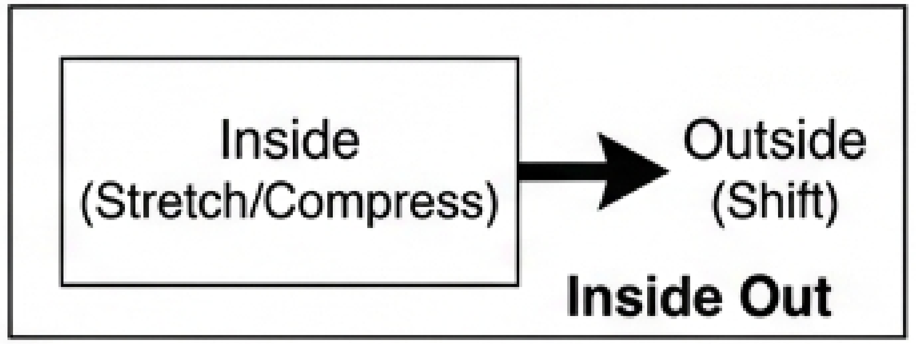
Thinking $f(2x)$ stretches the graph wider.

Correction (Bold):

Remember: Inside numbers are reciprocal. Big numbers **Compress**.

Sequence Errors

Shifting before stretching.

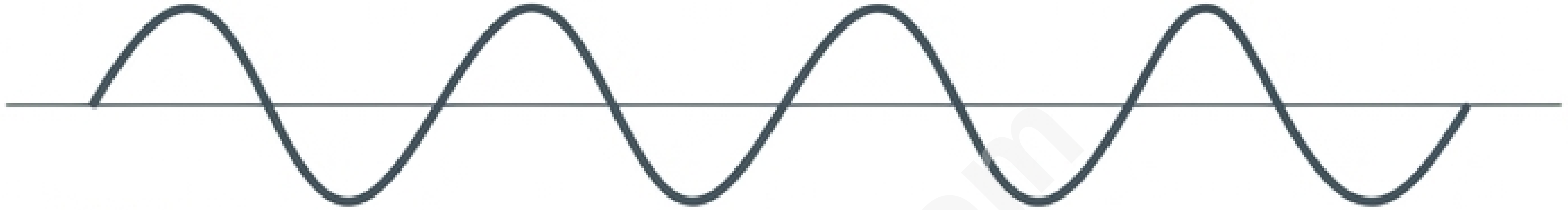


Correction (Bold):

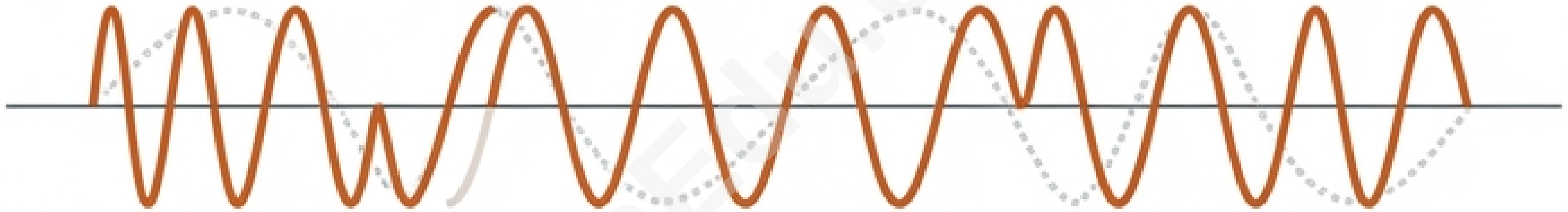
Remember: Always work **Inside Out**. Use a test point to verify.

Why This Matters: Real-World Applications

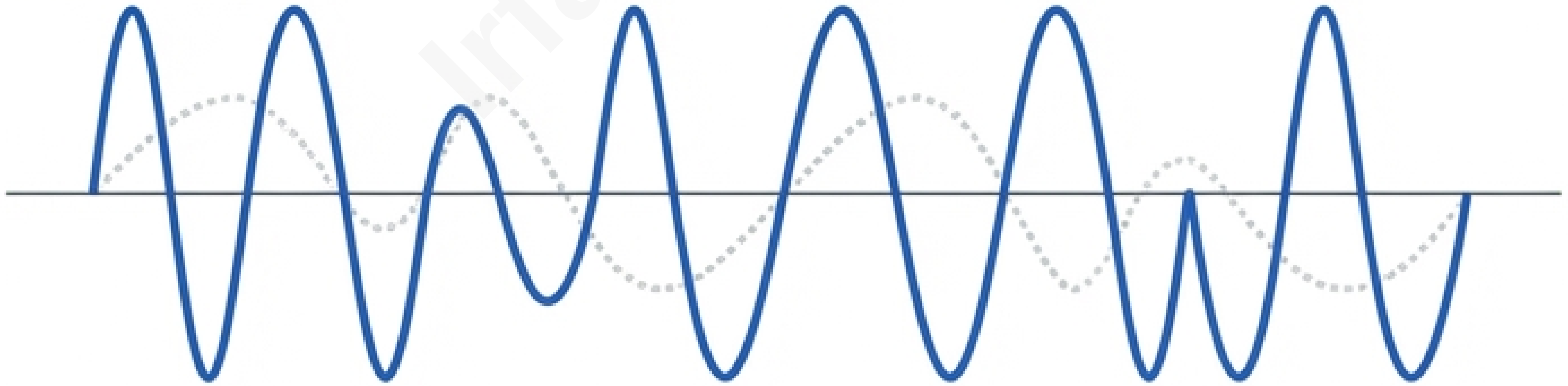
Standard Tone



Horizontal Compress = Higher Pitch



Vertical Stretch = Louder Volume



Context: From audio engineering to modeling inflation in economics, transformations allow us to predict dynamic changes in complex systems.

Summary Cheatsheet

Transformation Type	Vertical (Outside / y)	Horizontal (Inside / x)
Shift	$f(x) + k$ (Up/Down)	$f(x - h)$ (Right/Left)
Scale	$a \cdot f(x)$ (Stretch > 1)	$f(bx)$ (Compress > 1)
Reflect	$-f(x)$ (Over x-axis)	$f(-x)$ (Over y-axis)

"Identify the Parent. Apply the Rules (Inside First). Check your Points."