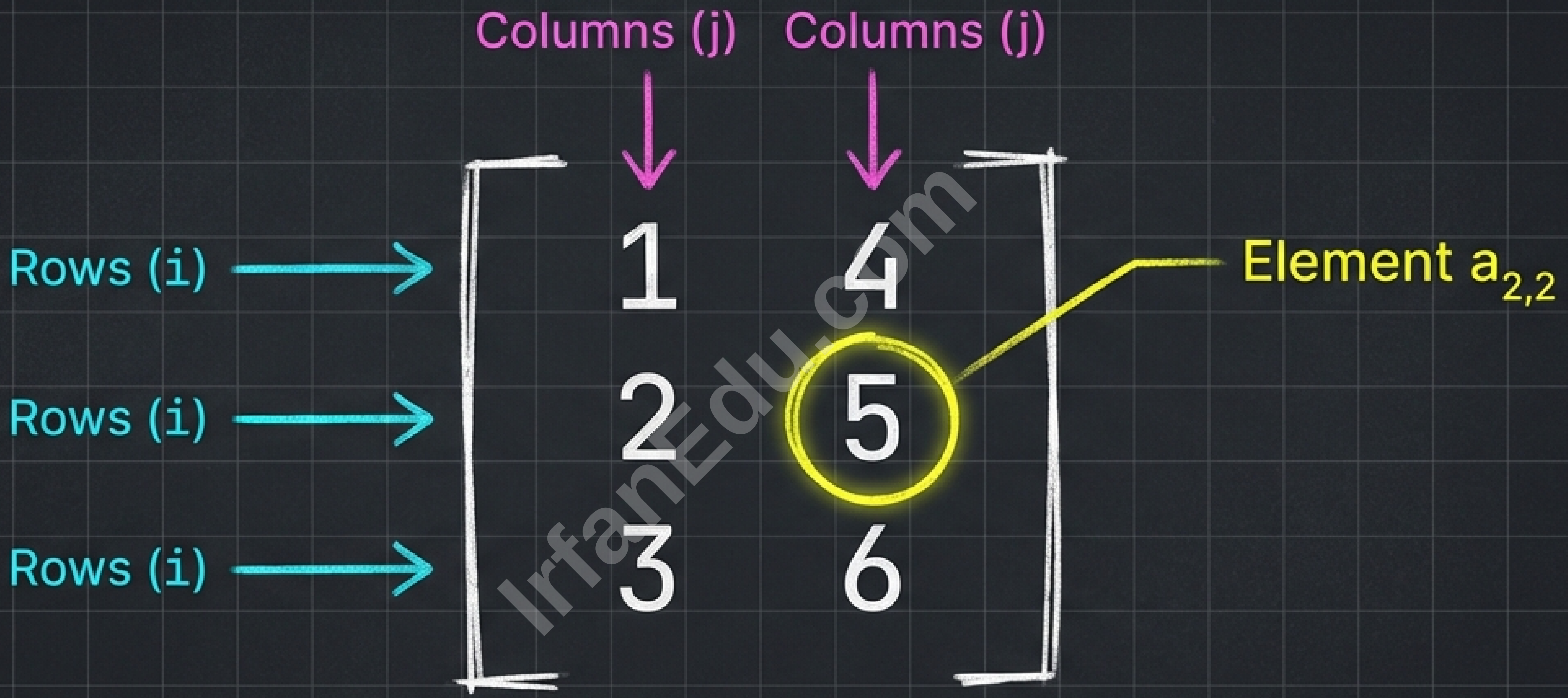
FOUNDATIONS OF MATRIX ALGEBRA

Decoding the Grid: From Atomic Elements to Complex Systems

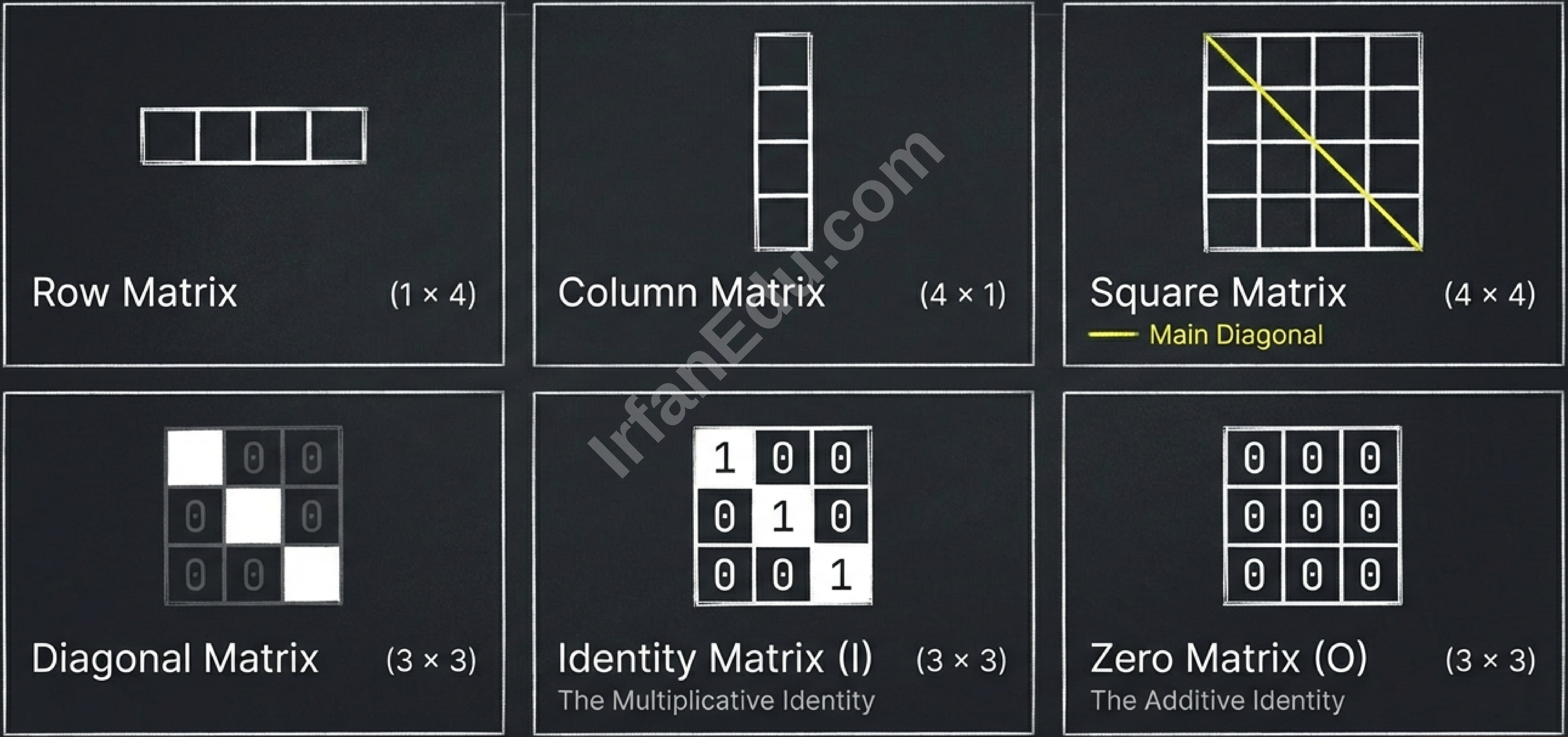
THE ANATOMY OF A MATRIX



Dimension = 3 Rows x 2 Columns

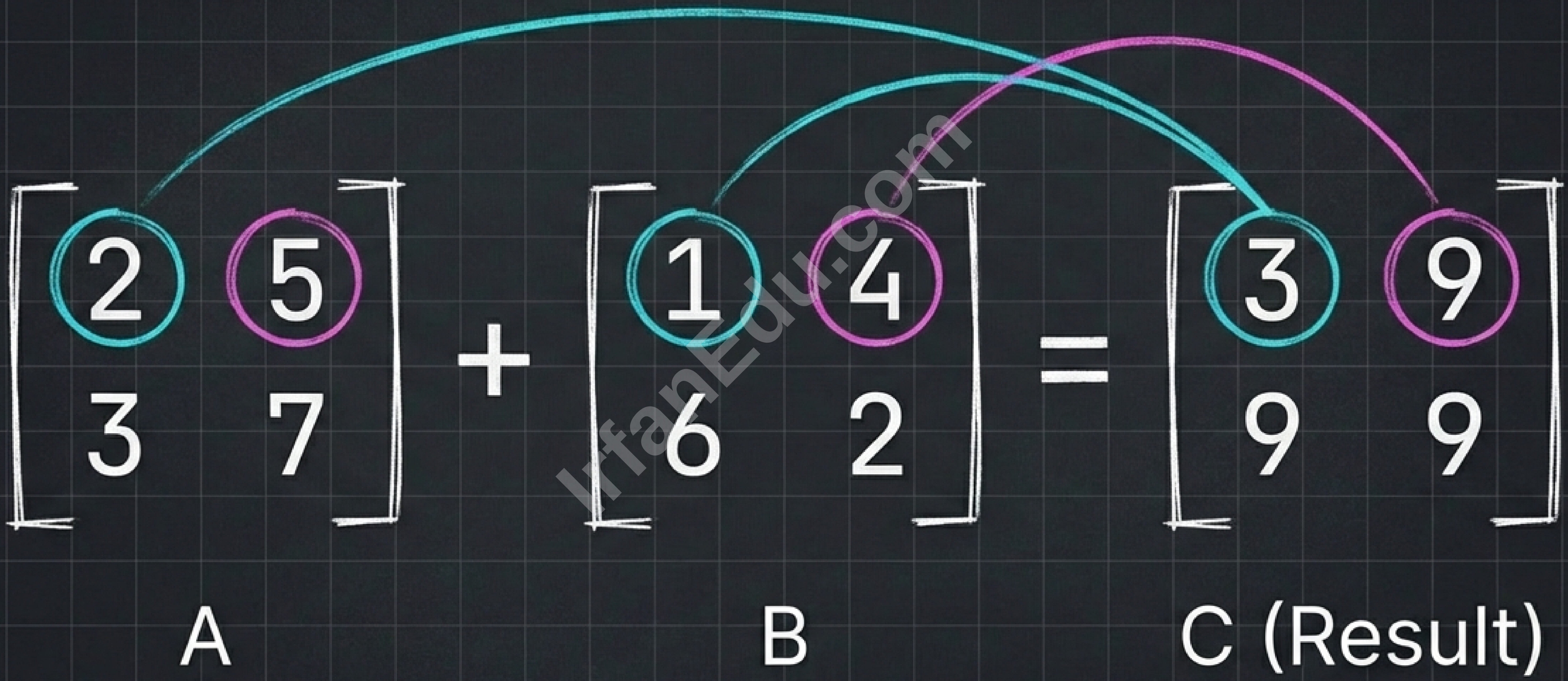
The address of any number is defined by row first, then column.

CLASSIFICATION BY STRUCTURE



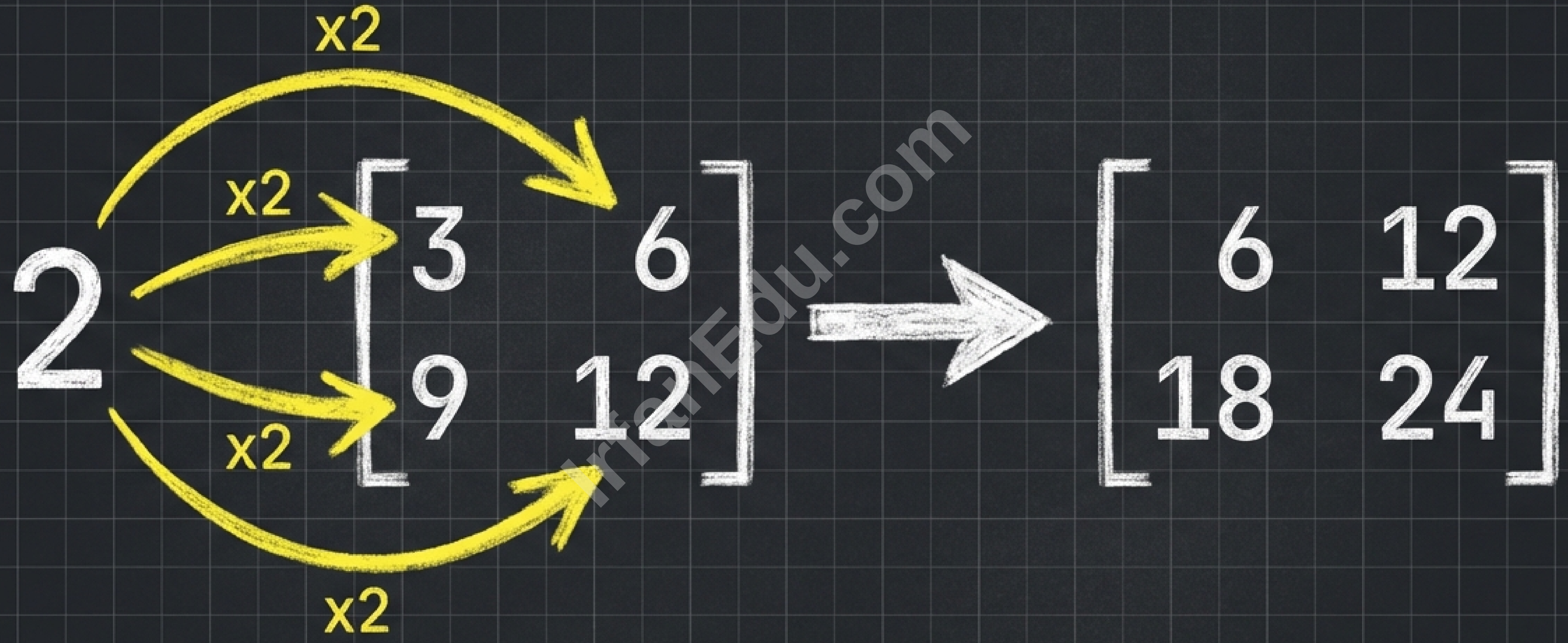
COMBINING FORCES (ADDITION)

Dimensions Must Match Exactly



$$\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 9 & 9 \end{bmatrix}$$

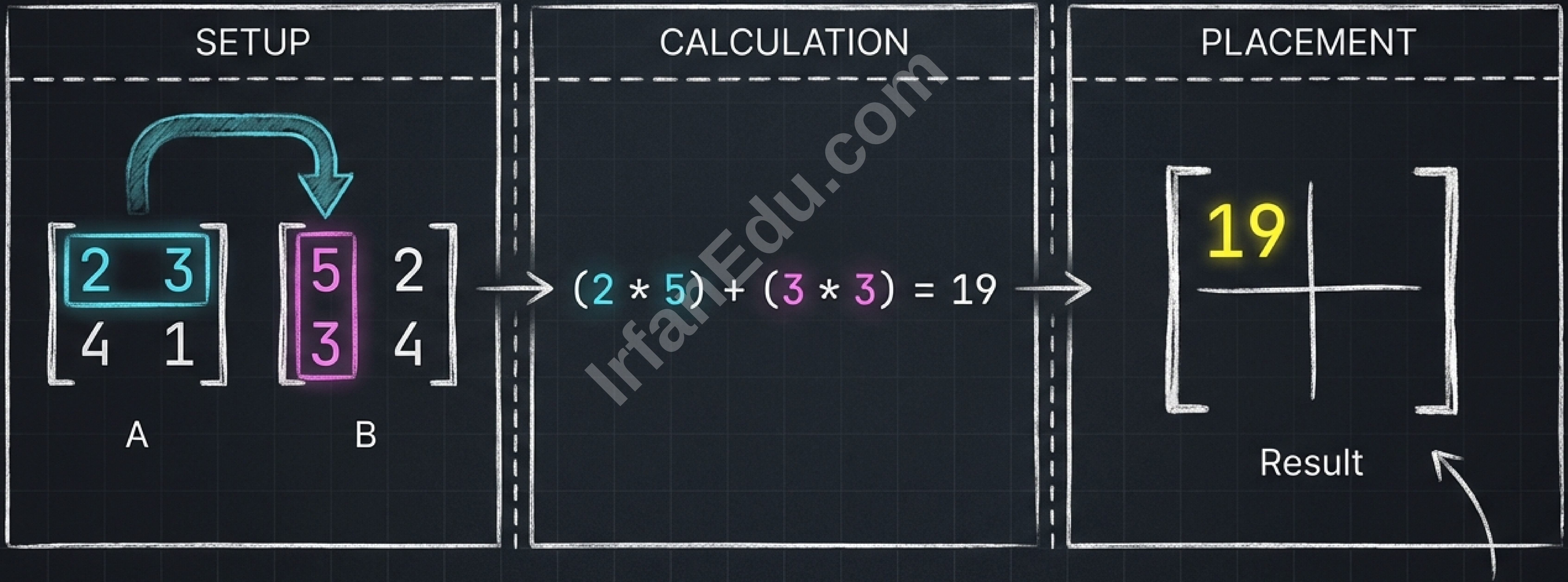
SCALING THE SYSTEM



The scalar k distributes into every single cell like a flood fill.

THE DOT PRODUCT MECHANISM

Row dives into Column



Repeat for every position.

RULES OF ENGAGEMENT

$$AB \neq BA$$

Matrix Multiplication is NOT Commutative. Order matters.

Must Match

$$[m \times n] \cdot [n \times p]$$

Resulting Dimension

Associative Property DOES hold: $(AB)C = A(BC)$

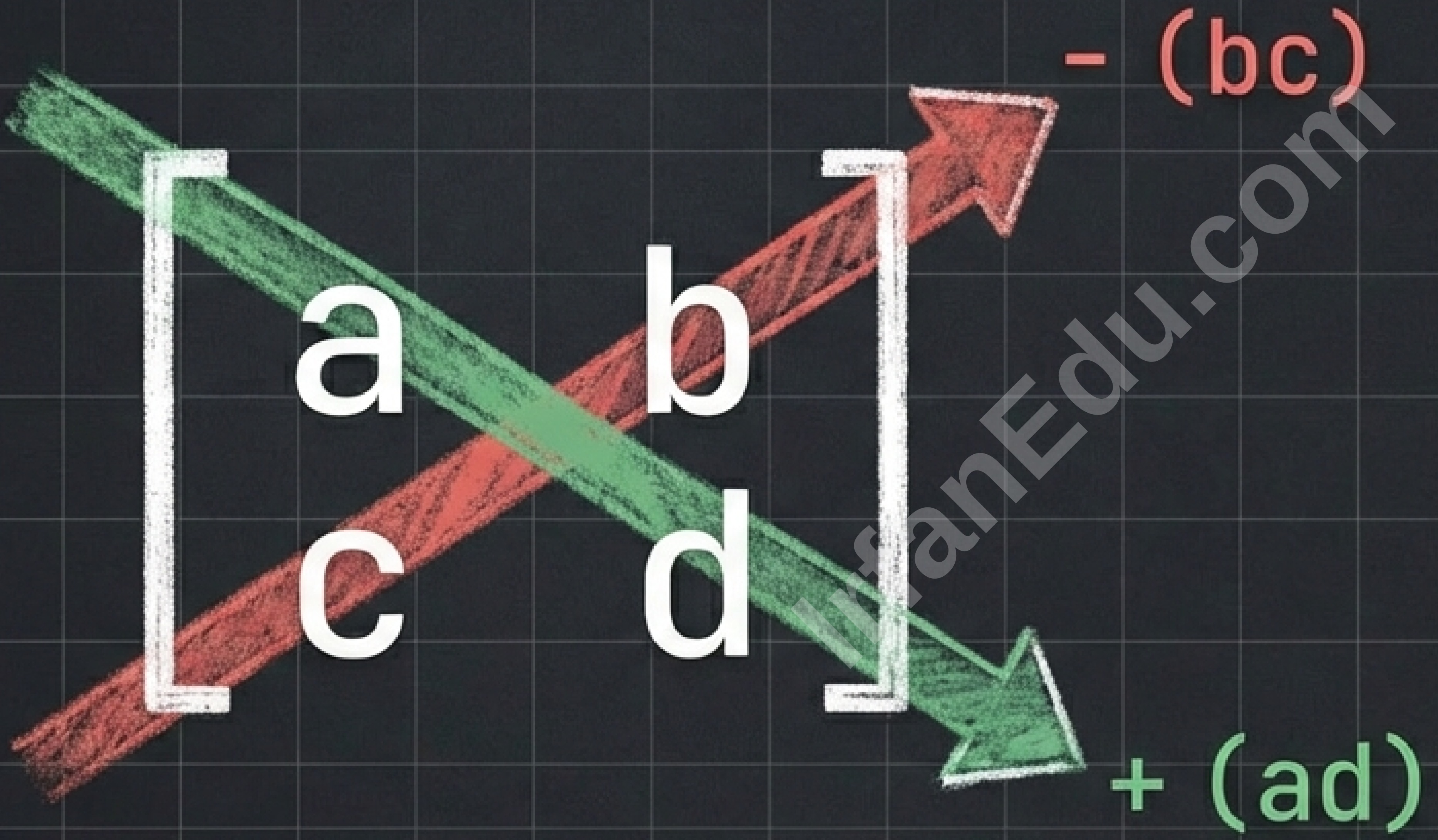
THE MIRROR EFFECT (TRANSPOSE)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

- Rows become Columns
- $(A^T)^T = A$
- Symmetric Matrix: $A = A^T$

THE DETERMINANT

The scalar DNA of a square matrix



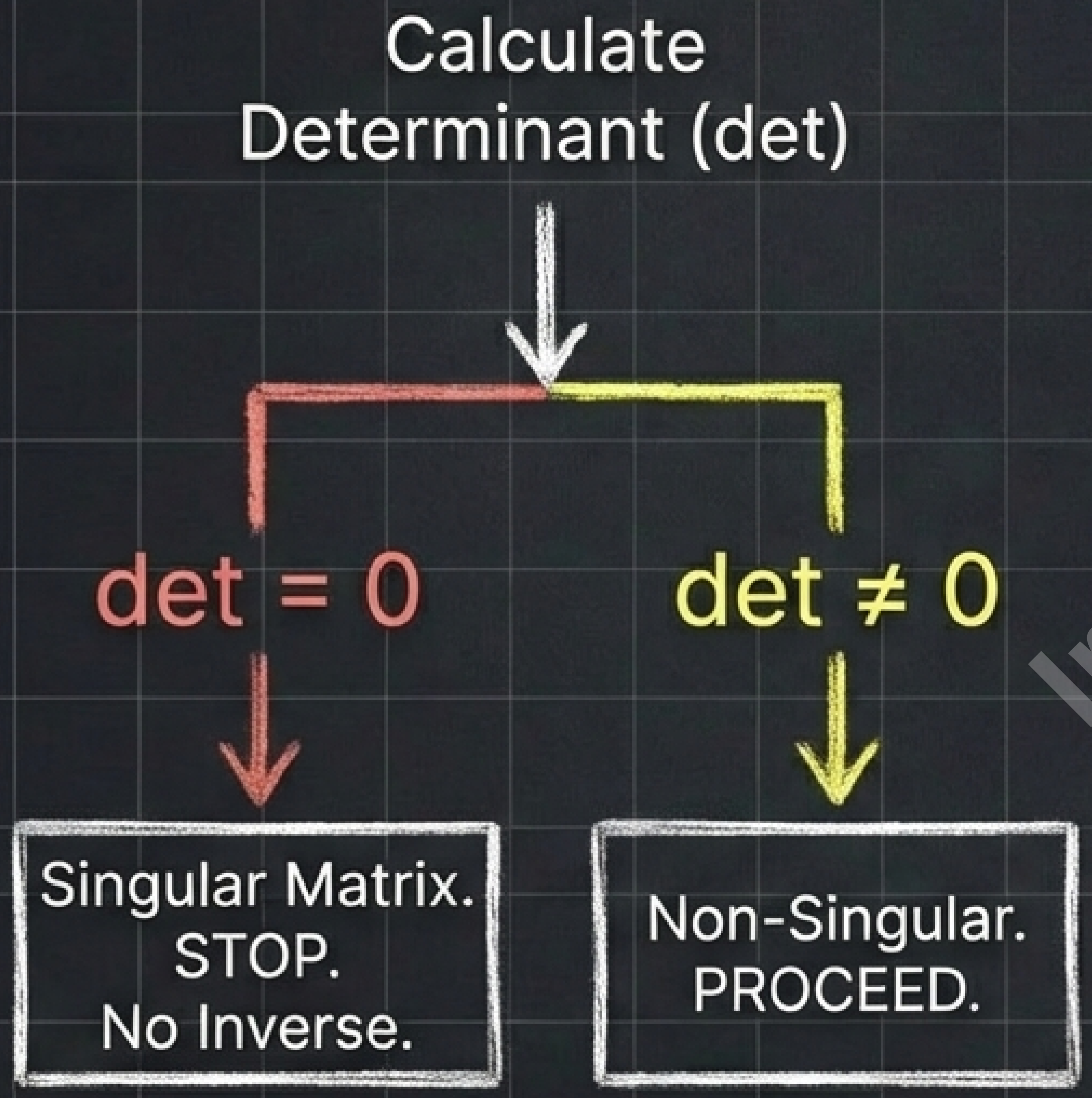
$$\det\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$$

$$= (3)(4) - (5)(2)$$

$$= 12 - 10 = 2$$

$$\det(A) = ad - bc$$

UNDOING THE MATRIX (THE INVERSE)



The Formula for 2x2 Inverse.

$$A^{-1} = \frac{1}{det} * [\text{Adjoint Matrix}]$$

Original:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow$$

Adjoint:

$$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

SOLVING LINEAR SYSTEMS

Traditional Algebra

$$2x + 3y = 8$$
$$4x + 1y = 10$$

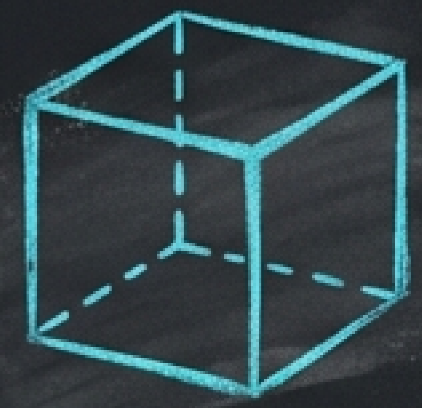
Matrix Notation (AX = B)

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 8 \\ 10 \end{bmatrix}$$

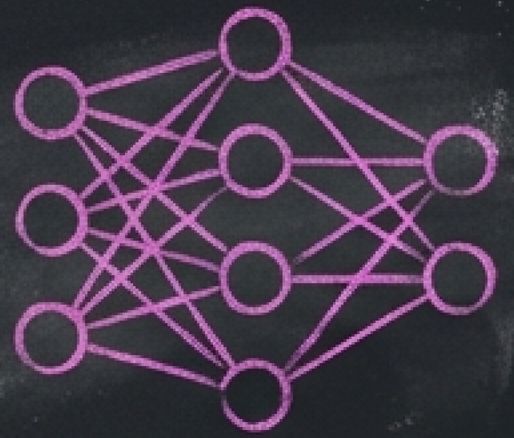
$$X = A^{-1} * B$$

Multiplication by inverse replaces division.

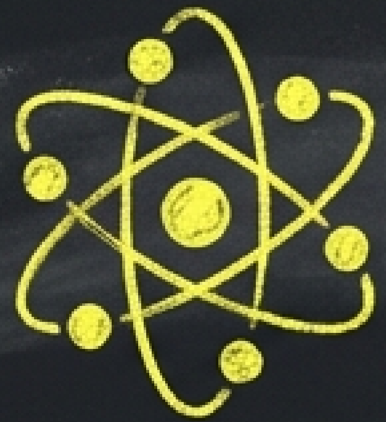
THE ENGINE OF MODERN TECH



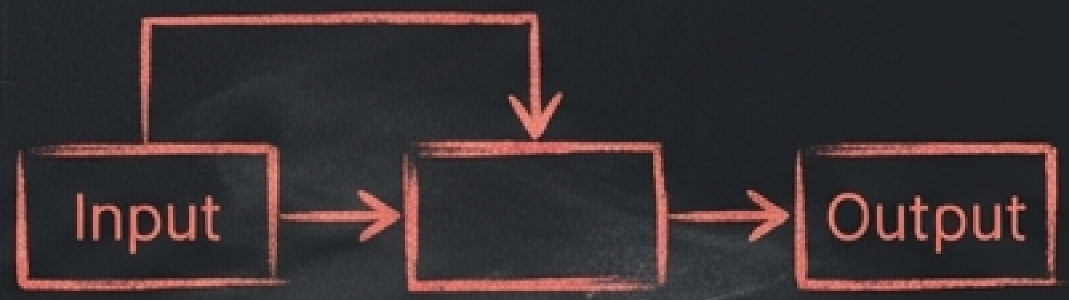
Matrices transform and rotate 3D coordinates.



Weights and inputs processed via massive matrix multiplication.



Representing quantum states and operators.



Input-output models for industry analysis.

AVOID THESE TRAPS

THE COMMUTATIVE ASSUMPTION

Thinking AB = BA.

DIMENSION MISMATCH

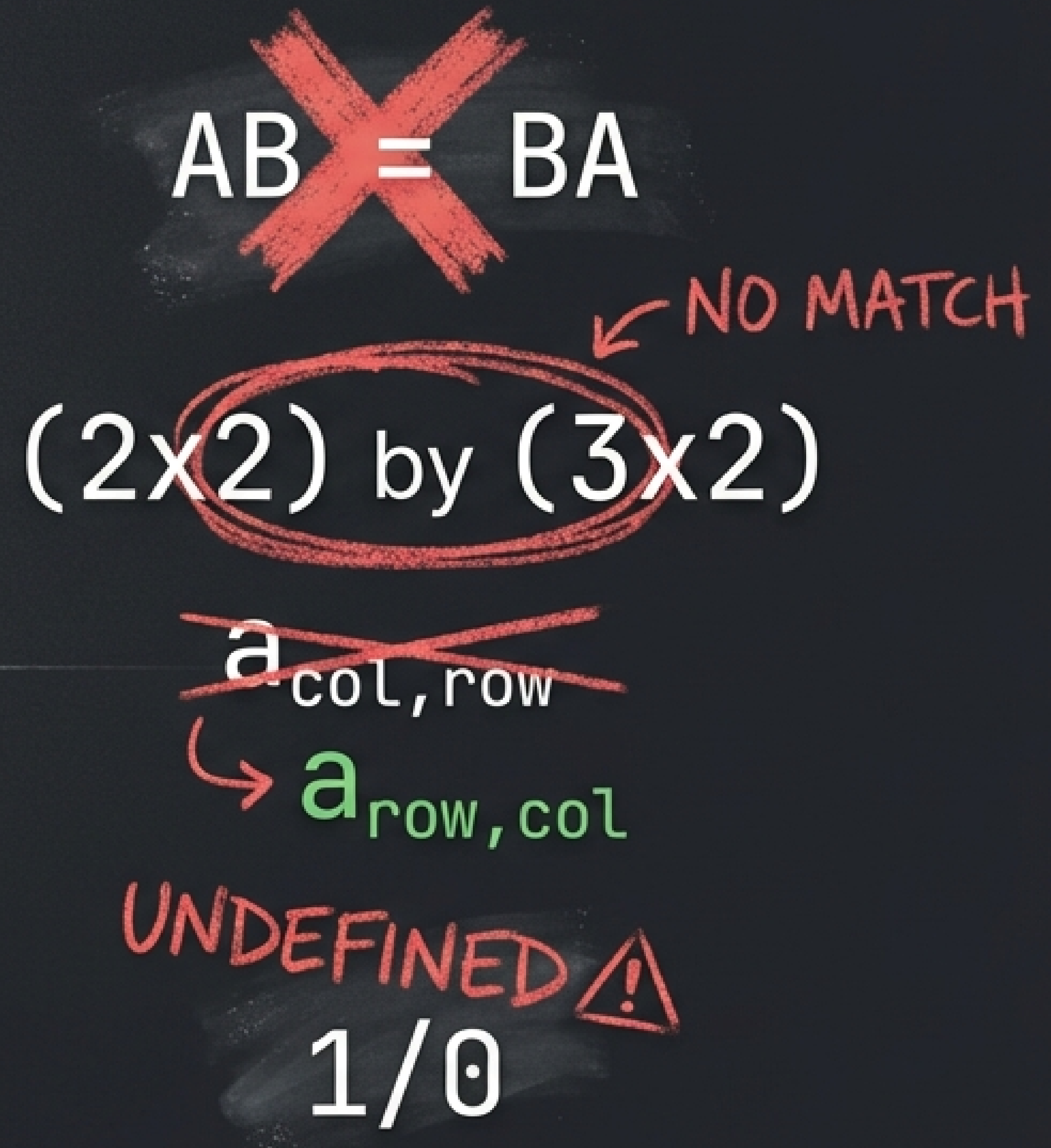
Multiplying (2x2) by (3x2).

INDEX CONFUSION

Mixing up row/column order.

THE SINGULAR TRAP

Inverting when Determinant is 0.



PRO TIP Always multiply your inverse by the original matrix to verify you get Identity (I).

TEST YOUR MASTERY

Find A + B

$$\begin{bmatrix} 5 & 2 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 6 & 2 \end{bmatrix}$$

Calculate 3A

$$3 \cdot \begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix}$$

Find Determinant

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

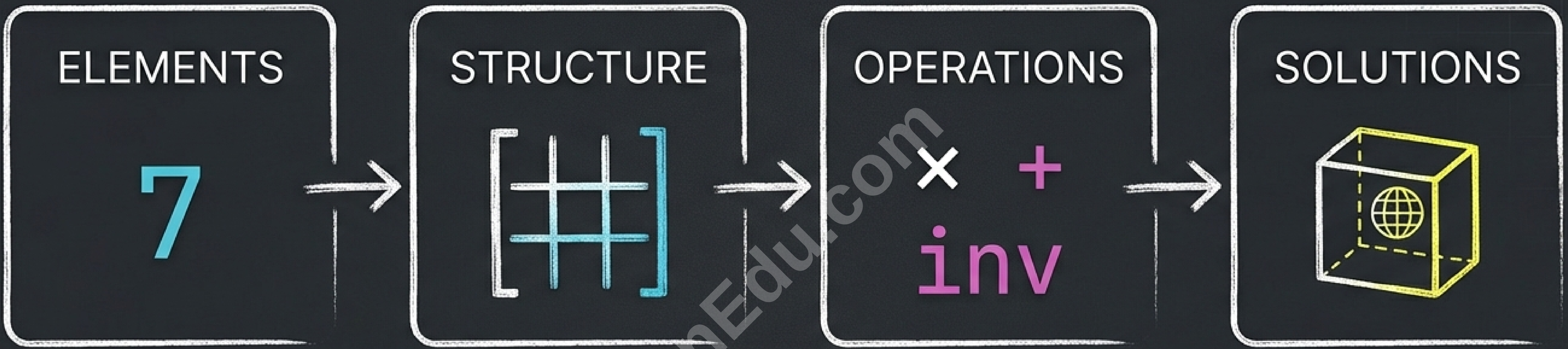
Find Inverse

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

ANSWERS:

1. $\begin{bmatrix} 6 & 6 \\ 9 & 9 \end{bmatrix}$
2. $\begin{bmatrix} 6 & -3 \\ 12 & 15 \end{bmatrix}$
3. 2
4. $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

THE UNIVERSAL LANGUAGE



"Mastering matrices is about seeing the structure behind the data. Whether in physics, AI, or engineering, the grid is the foundation."

Keep practicing. Verify dimensions. Trust the logic.