Mastering Quadratic & Absolute Value Inequalities

$|x|$

A Deconstructed Workshop

Breaking down complex expressions into manageable steps.

The Antagonist: Quadratics

Definition: Comparing a quadratic expression ($ax^2 + bx + c$) to zero or another value.

Goal: Find all x-values that make the inequality true.

The Antagonist: Absolute Value

Definition: Represents the distance a number sits from zero.

Key Concept: Distance is always positive or zero; never negative.

Module A: The 4-Step Roadmap

Step 1: Rearrange
Move terms to one side.

→

Step 2: Boundary Points
Find the roots/zeros.

→

Step 3: Test Intervals
Check regions on the number line.

→

Step 4: Solution Set
Identify satisfying intervals.

The Setup: Rearrange & Identify

Step 1: Rearrange so zero is on one side.

Step 2: Find Boundary Points by solving where expression = 0.

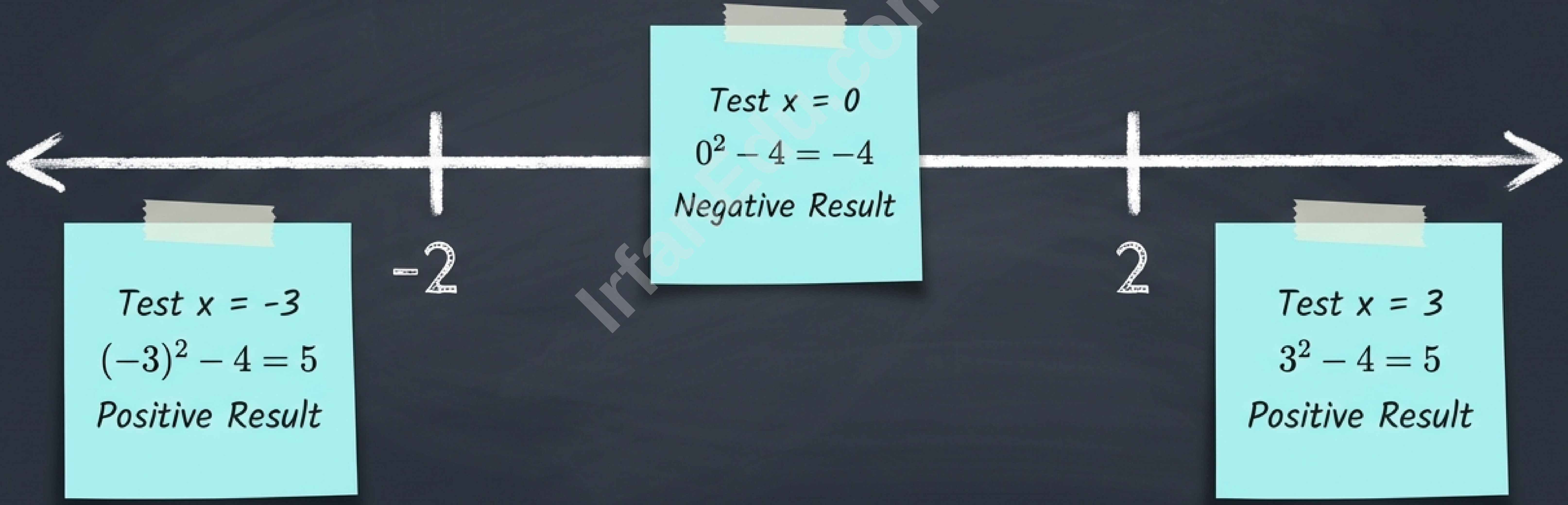
Inequality: $x^2 - 4 > 0$

Equation: $x^2 - 4 = 0$

Factors: $(x - 2)(x + 2) = 0$

Roots: $x = 2$ and $x = -2$

Step 3: The Number Line Test



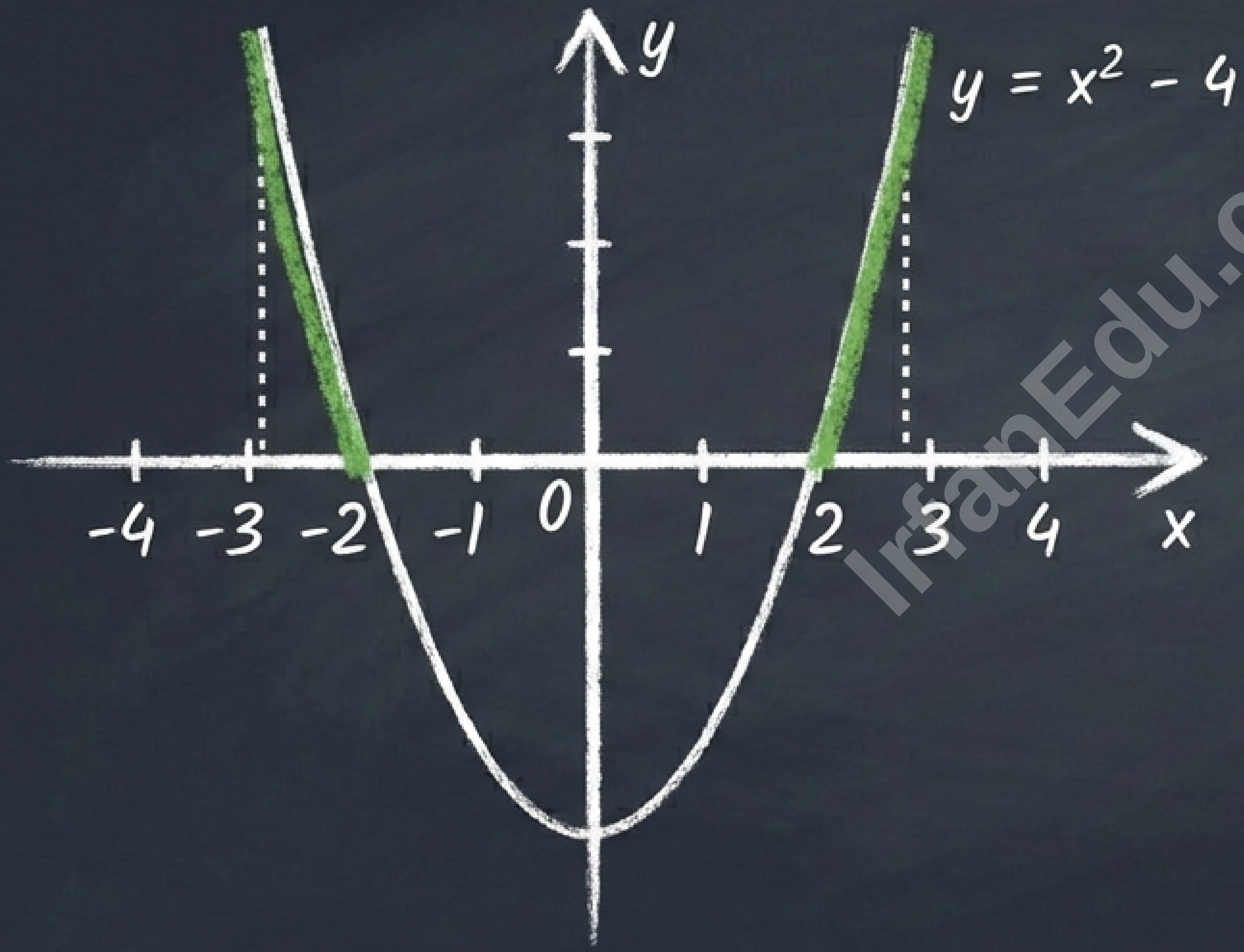
Step 4: Determine the Solution Set

Since $x^2 - 4 > 0$, we select the Positive regions.

Final Answer: $(-\infty, -2) \cup (2, \infty)$

Tip: Use Union symbol (∪) to combine separate intervals.

Visualizing Graphically



The solution corresponds to where the graph sits ABOVE the x-axis ($y > 0$).

Intersection points mark the boundary values.

Module B: Absolute Value Patterns

Concept:
Absolute Value = Distance from Zero.

When solving inequalities, we consider two scenarios based on the direction of the inequality sign.

The "Less Than" Pattern ($|x| < a$)

Rule:
Translate to Compound Inequality
$-a < x < a$

Example: $|x - 3| \leq 5$

Becomes: $-5 \leq x - 3 \leq 5$

Simplify: $-2 \leq x \leq 8$

The 'AND' Sandwich

The "Greater Than" Pattern ($|x| > a$)

Rule: Split into Separate Conditions

$x > a$ OR $x < -a$

Example: $|x+2| > 4$

Split: $x + 2 > 4$ OR $x + 2 < -4$

Result: $x > 2$ OR $x < -6$

The "OR" Split

The Danger Zone: Essential Tips

Watch Your Signs

The inequality symbol FLIPS direction when multiplying or dividing by a negative.

Isolate First!

Always isolate the absolute value expression before applying any rules.

Notation Matters

Use brackets [ ] for "or equal to".

Use parentheses ( ) for strict inequalities.

Recognizing Special Cases

The Impossible

$|x| <$ Negative Number

Distance cannot be negative.
Result: NO SOLUTION

The Universal

$|x| >$ Negative Number

Distance is always > negative.
Result: ALL REAL NUMBERS

Real-World Applications

Engineering: Tolerance Ranges

Science: Measurement Uncertainty

Business: Profit Optimization

Building Proficiency

Start Simple

Begin with basic problems.

Verify

Substitute test values back into the original inequality.

Practice

Gradually increase complexity.

Understanding inequalities isn't just about algebra. It develops analytical thinking skills, logical reasoning, and problem-solving capabilities for everyday life.